The Devil is in the Tails

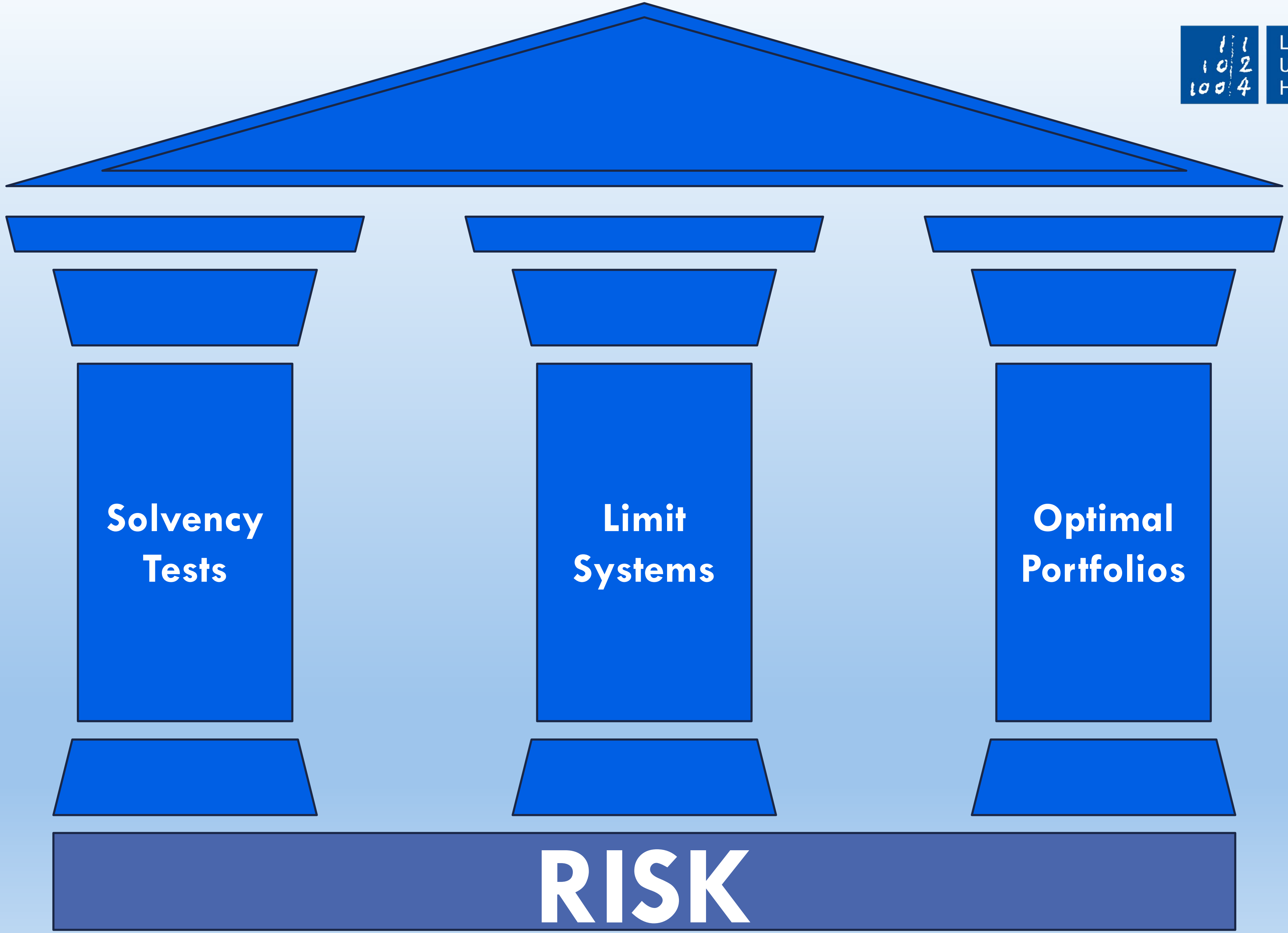
Justin Plückebaum

Leibniz Universität Hannover

www.insurance.uni-hannover.de

House of Insurance Day 2023 – November 9, 2023

Leibniz Universität Hannover



Leibniz Universität Hannover

Solvency Tests

- **Solvency balance sheet**

Assets	Liabilities
A_t	L_t
	$E_t = A_t - L_t$

  - The quantities at time $t = 0$ are known, the quantities at time $t = 1$ are random variables on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$.
  - The increment of the net asset value is $\Delta E_1 := E_1 - E_0$.

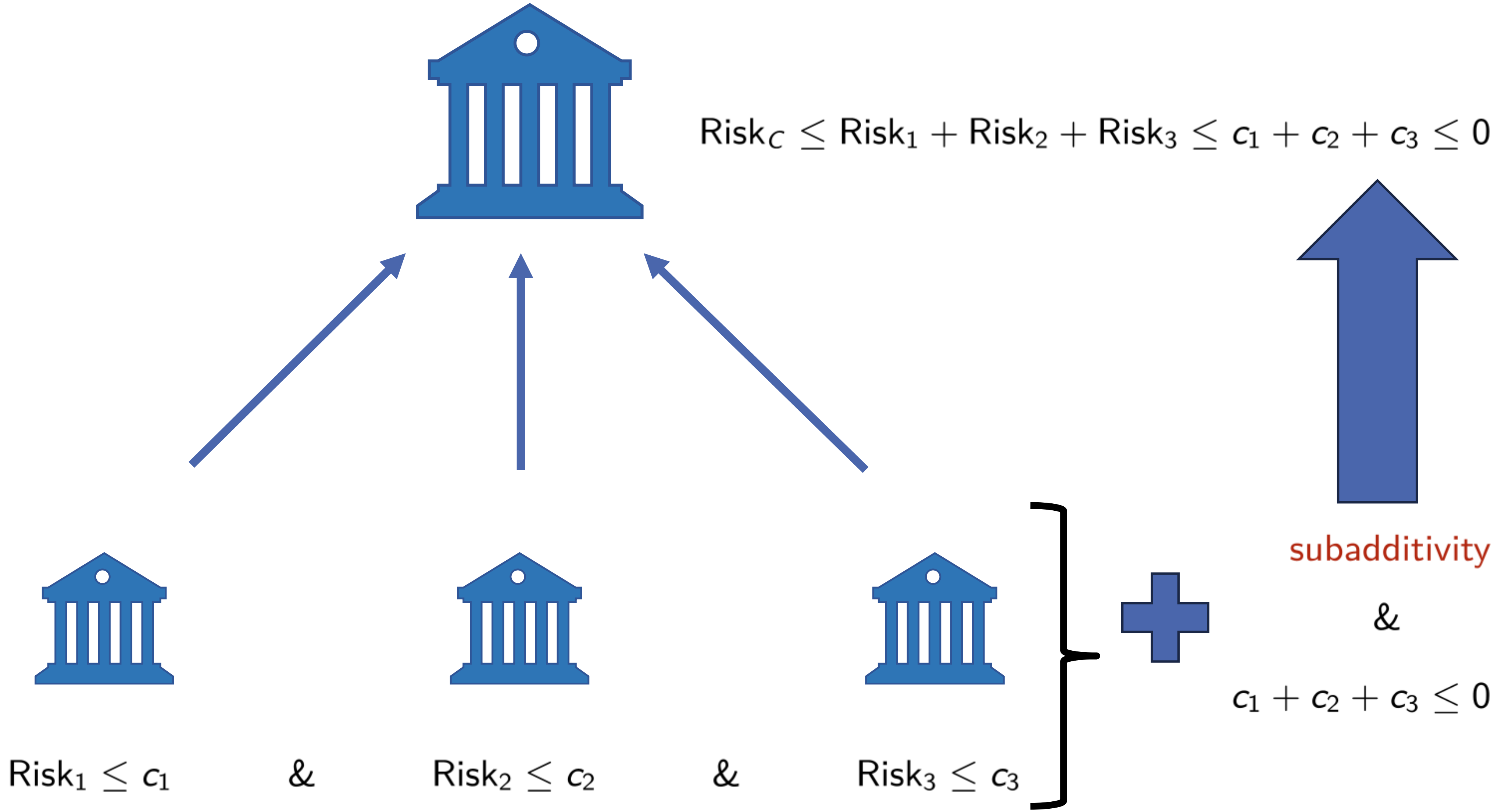
- **Solvency test**

  For a given regulatory monetary risk measure ρ, the company is **solvent**[1] if

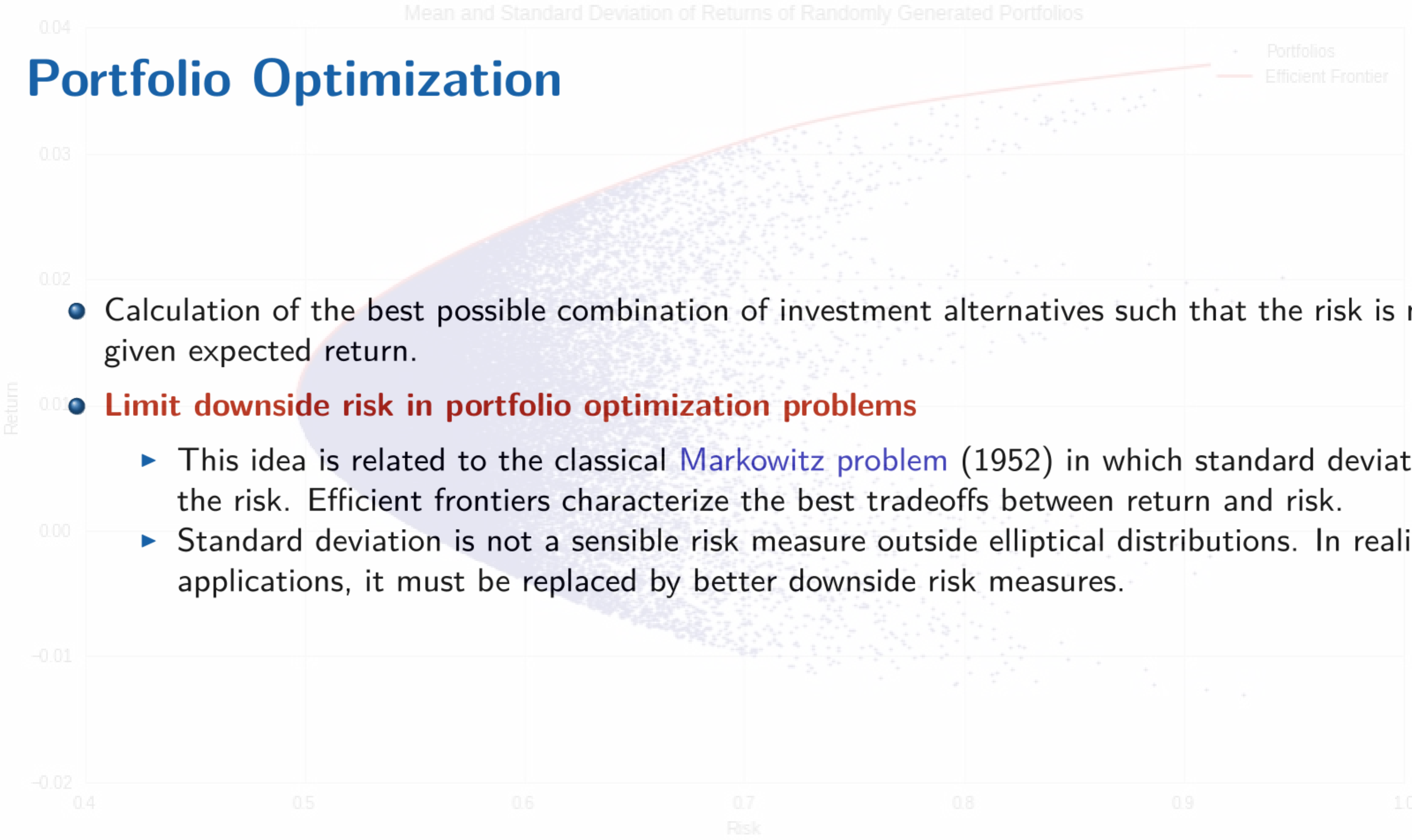
$$\rho(\Delta E_1) \leq E_0 \iff \rho(E_1) \leq 0 \iff E_1 \in \mathcal{A}_\rho$$

[1] In practice, solvency capital requirements may only refer to "unexpected" losses. In this case, in the definition of ΔE_1, E_0 is replaced by the expected value of (the suitably discounted) E_1. In this respect, the European regulatory framework for insurance companies Solvency II is self-contradictory.

Limit Systems



Portfolio Optimization



- Calculation of the best possible combination of investment alternatives such that the risk is minimized for a given expected return.
- **Limit downside risk in portfolio optimization problems**
  - This idea is related to the classical Markowitz problem (1952) in which standard deviation quantifies the risk. Efficient frontiers characterize the best tradeoffs between return and risk.
  - Standard deviation is not a sensible risk measure outside elliptical distributions. In realistic applications, it must be replaced by better downside risk measures.

Portfolio Optimization

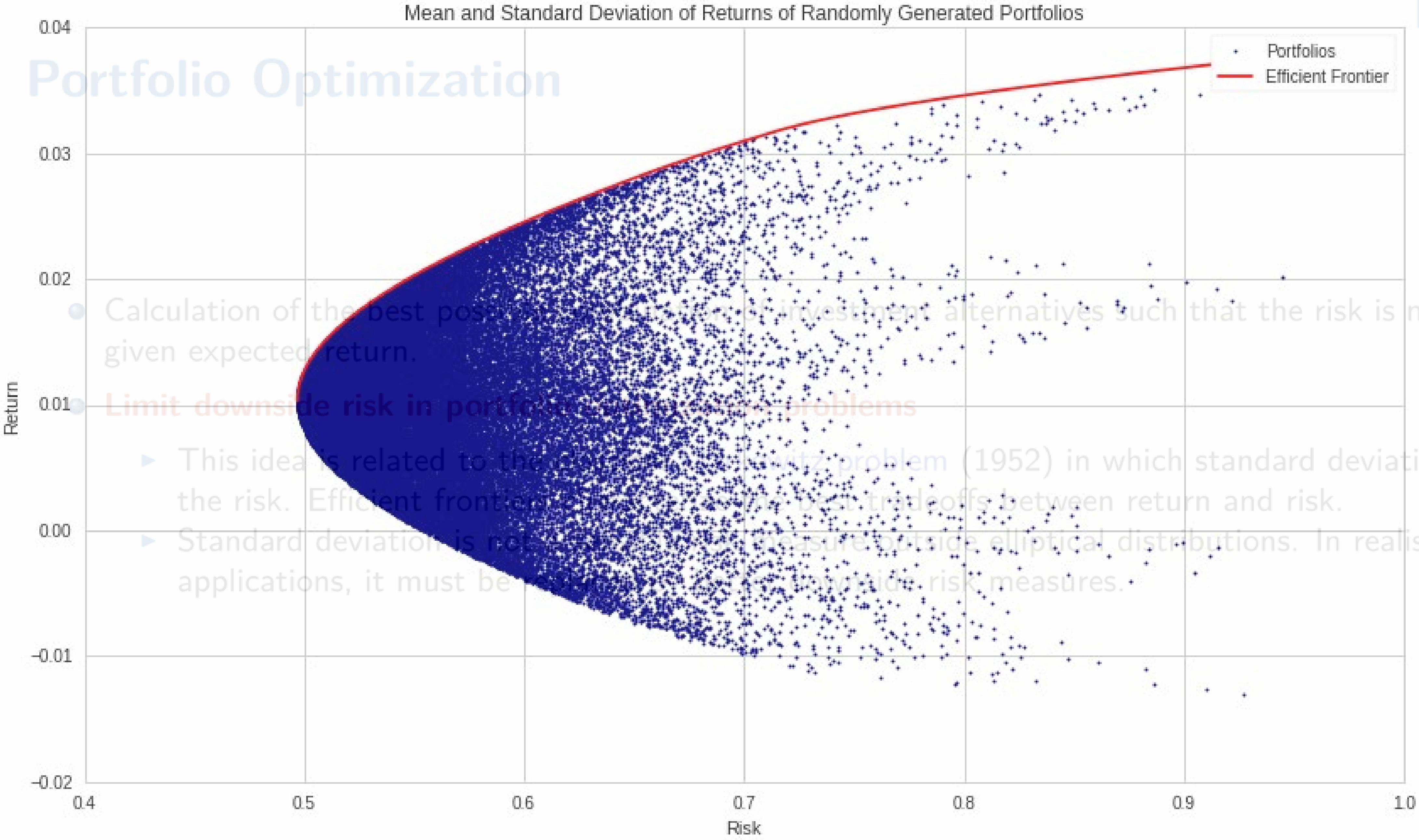


Image Credits: Abasi, Margenot and Granizo-Mackenzie, 2020

How Do We Measure Risk?

- In general, a risk measure is a functional

$$\rho : \mathcal{X} \to \mathbb{R},\ X \mapsto \rho(X),$$

which quantifies the risk $\rho(X)$ of a financial position X.

- For monetary risk measures:
  - A financial position $X \in \mathcal{X}$ is said to be acceptable with respect to a given monetary risk measure ρ if $\rho(X) \leq 0$.
  - ρ can be interpreted as capital requirement: smallest amount of money that must be added to X to become acceptable.

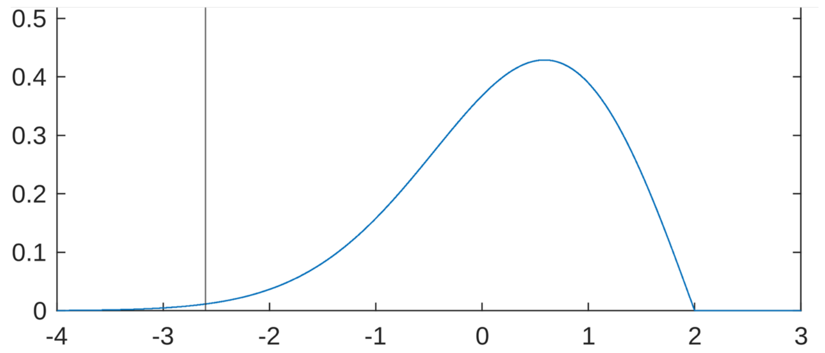
Value at Risk

Value at Risk

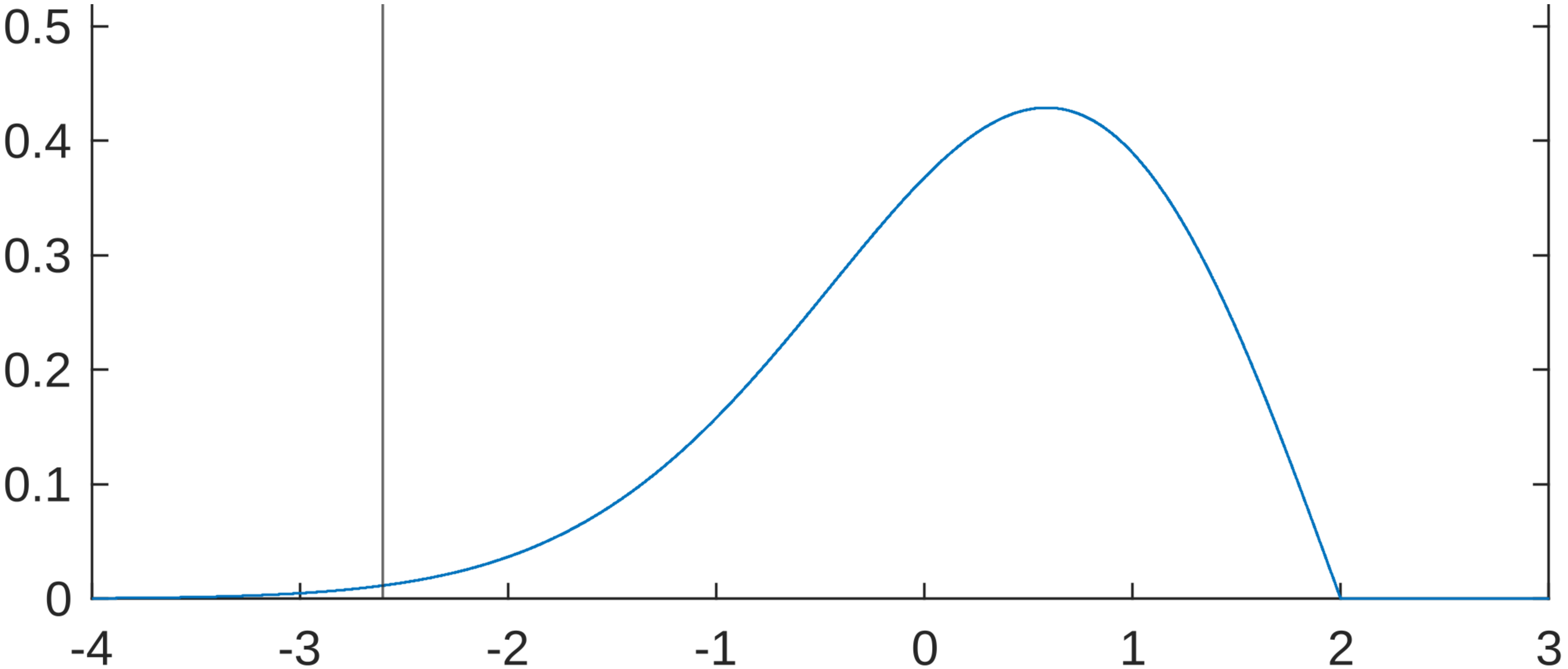
Value at Risk at level $\alpha \in (0,1)$ of a financial position X:

$$V@R_\alpha(X) := \inf\{m \in \mathbb{R} : P[X + m < 0] \leq \alpha\}$$

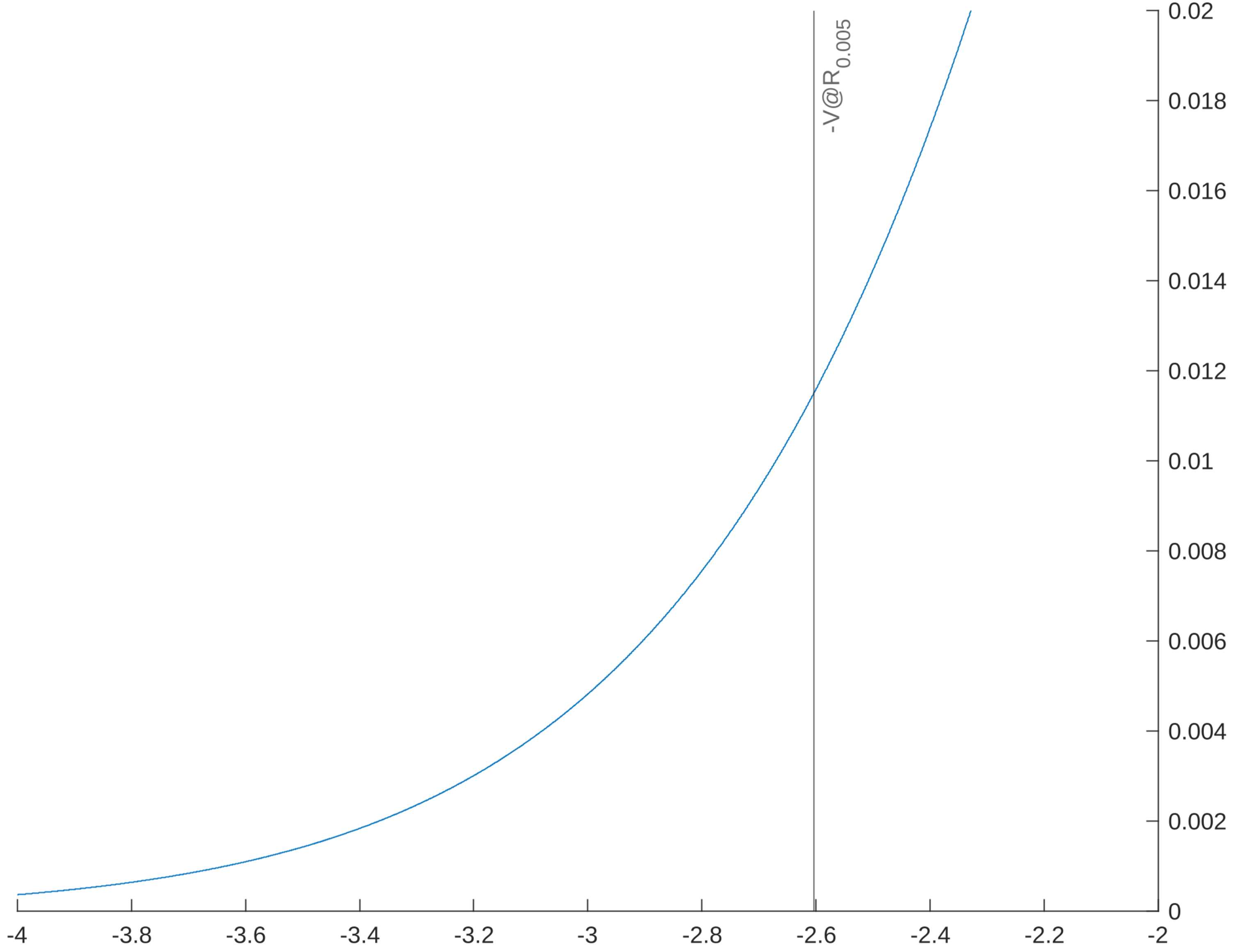
- $V@R_\alpha(X)$ is the smallest monetary amount that needs to be added to X such that the probability of a loss becomes smaller than α (capital requirement).
- Value at Risk has two serious deficiencies:
  - V@R neglects extreme events that occur with small probability.
  - V@R does not generally reward diversification, but charges a larger risk amount for a diversified position in many cases (no sub-additivity).
  - **Solvency II** V@R at level $\alpha = 0.5\%$



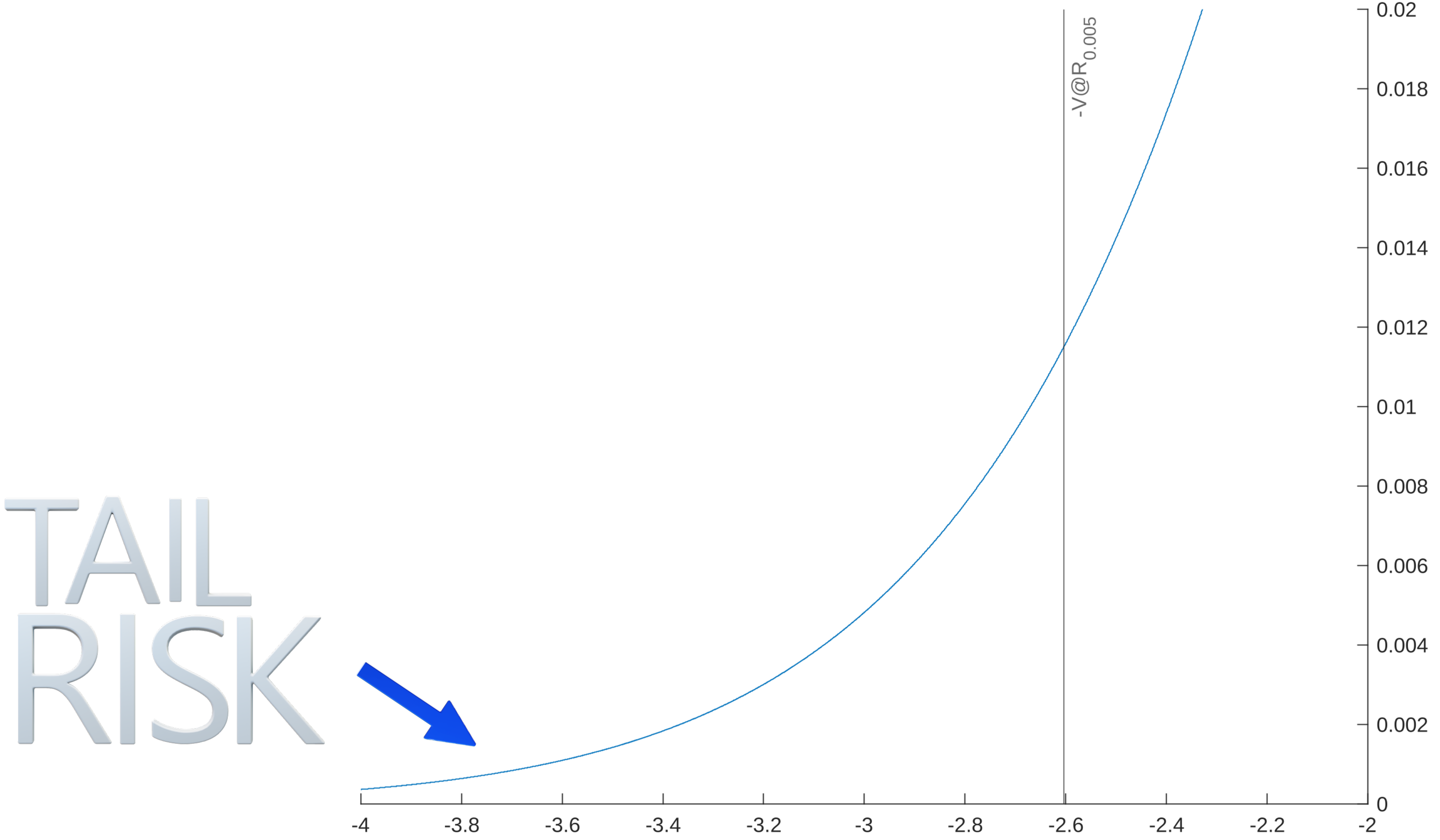
Value at Risk



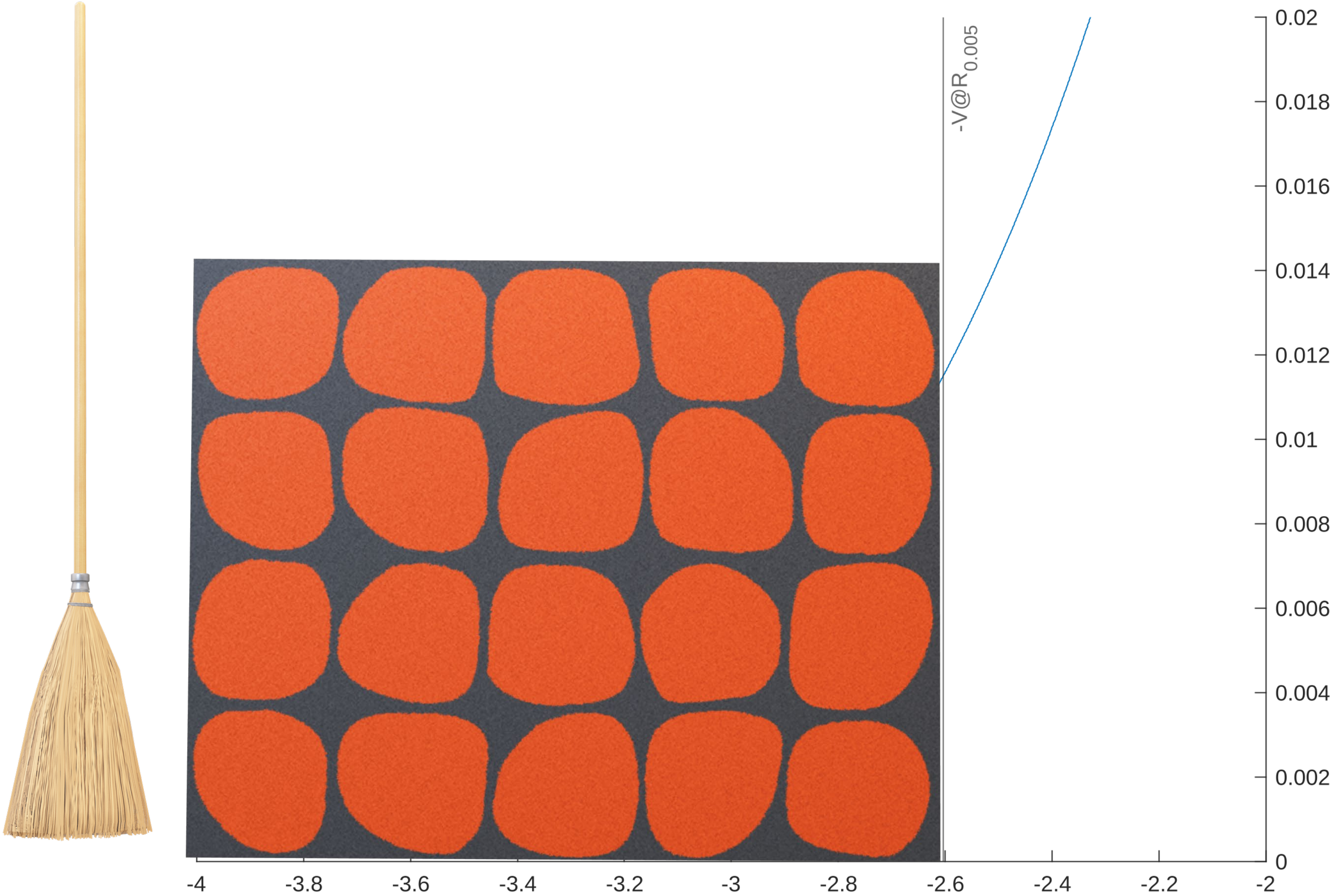
Value at Risk



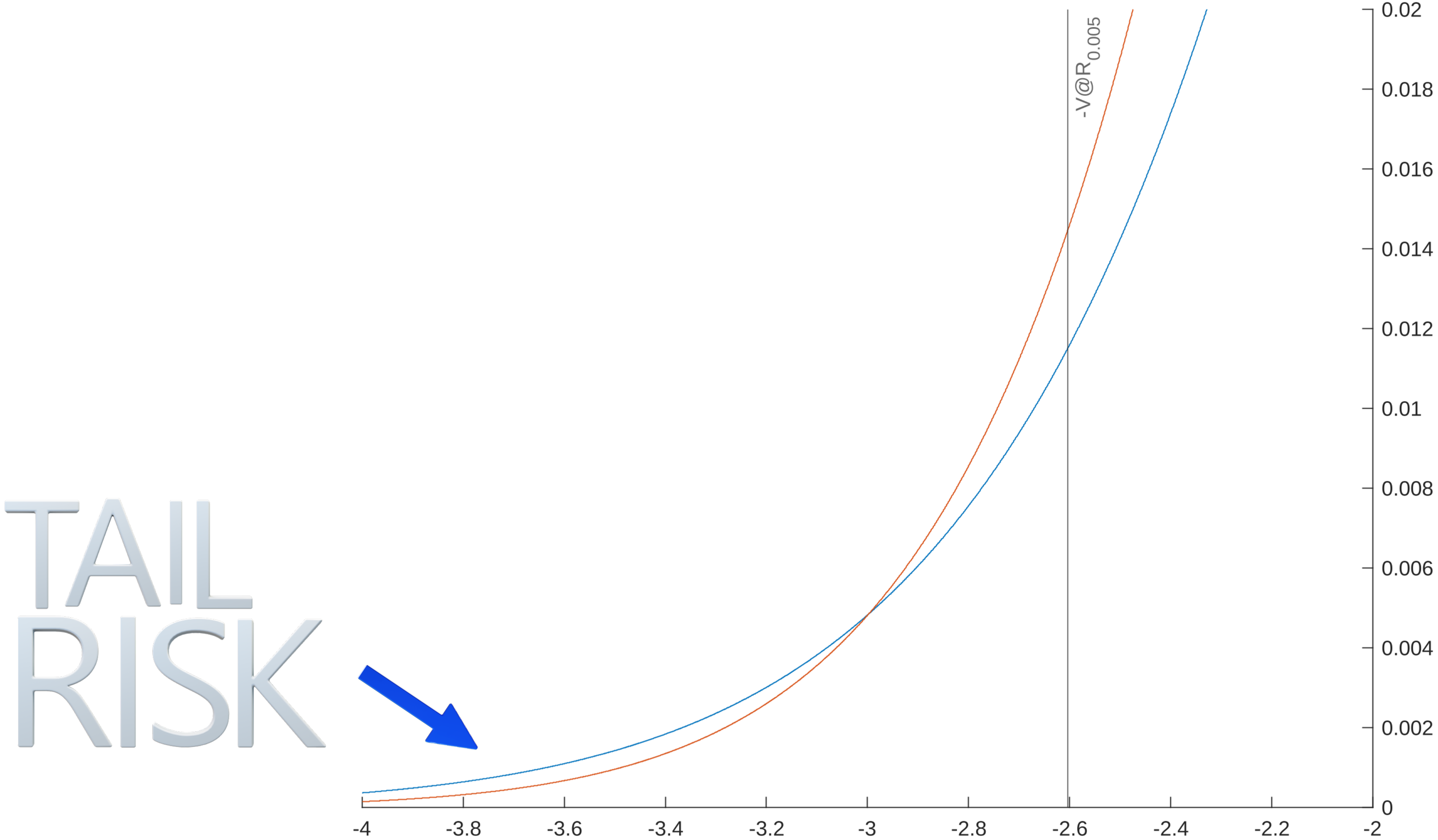
Value at Risk



Value at Risk



Value at Risk



Average Value at Risk

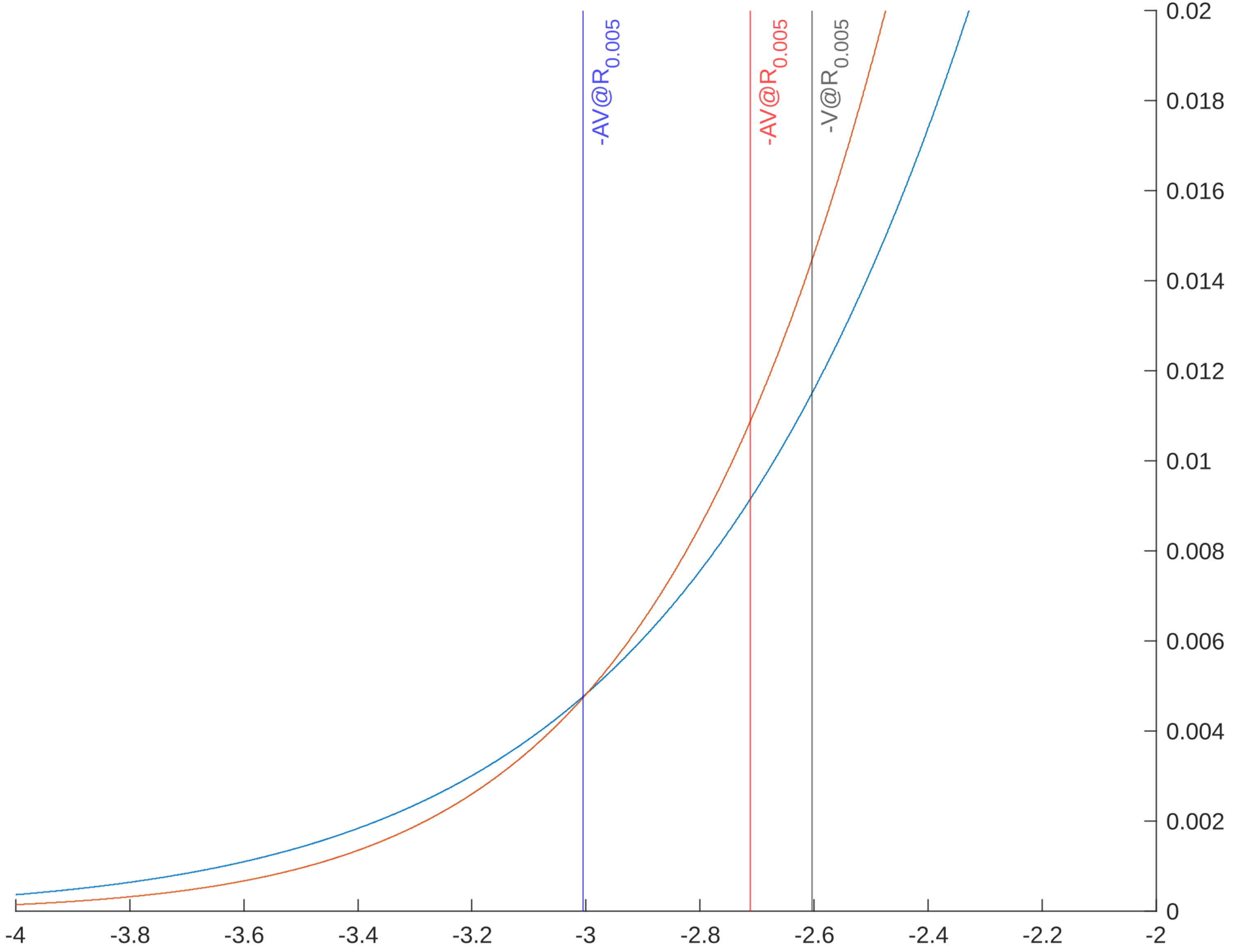
Average Value at Risk

Average Value at Risk at level $\alpha \in (0,1)$ of a financial position X:

$$\mathrm{AV@R}_\alpha(X) := \frac{1}{\alpha}\int_0^\alpha \mathrm{V@R}_\beta(X)d\beta$$

- More conservative than the Value at Risk for the same level α.
- Average loss of losses below the α-quantile.
- **Regulatory standards**
  - **Swiss solvency test** AV@R at level $\alpha = 1\%$
  - **Basel III** AV@R with level $\alpha = 2.5\%$

Average Value at Risk



Recovery of Claims – Stylized ALM-Model

- Probability space $\Omega = \{g, b\}$ with $\mathbb{P}(b) = \frac{\alpha}{2}$ with $\alpha \approx 0$, say $\alpha = 0.5\%$ or $\alpha = 1\%$.
- Liabilities

$$L_1(\omega) = \begin{cases} 1 & \text{if } \omega = g, \\ 100 & \text{if } \omega = b. \end{cases}$$

- The company can manage its assets by engaging in a stylized financial contract with zero initial cost transferring dollars from the good state to the bad state.
- More specifically, we assume that the company can choose one of the following asset profiles at time 1:

$$A_1^k(\omega) = \begin{cases} 101 - k & \text{if } \omega = g, \\ k & \text{if } \omega = b, \end{cases} \quad \text{with} \quad k \in [0, 100].$$

- Hedging its liabilities completely would require the company to choose $k = 100$.

Recovery of Claims – Stylized ALM-Model (2)

- For any $k \in [0, 100]$, the company's net asset value is given by

$$E_1^k(\omega) = \begin{cases} 100 - k & \text{if } \omega = g, \\ k - 100 & \text{if } \omega = b. \end{cases}$$

- Due to limited liability, the corresponding shareholder value is

$$\max\{E_1^k(\omega), 0\} = \begin{cases} 100 - k & \text{if } \omega = g, \\ 0 & \text{if } \omega = b. \end{cases}$$

- Hence, the choice $k = 0$ is optimal from the perspective of shareholders – corresponding to **no recovery in the bad state**.
- The company is solvent independently of k:

$$\text{V@R}_\alpha(E_1^k) = k - 100 \leq 0, \quad \text{AV@R}_\alpha(E_1^k) = \frac{1}{\alpha}\left(\frac{\alpha}{2}(100 - k) + \frac{\alpha}{2}(k - 100)\right) = 0.$$

Motivation for a New Risk Measure

- The current standard risk measures Value at Risk and Average Value at Risk limit the probability of default of the subentity

$$P(A_1^1 \geq L_1^1) \ \geq \ 1 - \alpha,$$

but fail to control the size of recovery.

Recovery of Claims

The event

$$\{A_1^1 \geq \lambda L_1^1\}$$

contains those scenarios **for which a fraction of at least $\lambda \in [0, 1]$ of the claims is recovered**.

- If $\lambda < 1$, the probability $P(A_1 \geq \lambda L_1)$ of recovering fractions of at least $\lambda \in [0, 1]$ should be higher than $1 - \alpha$.
- Munari,Weber & Wilhelmy (2023) resolve this failure by developing novel recovery risk measures.

Recovery Value at Risk

Task

- Construct risk measures that control the probabilities $P(A_1 \geq \lambda L_1)$ of recovering a fraction of at least $\lambda \in [0,1]$ of the claims.

Definition

The **Recovery Value at Risk** with increasing *level function* $\gamma : [0,1] \to [0,1)$ is defined by

$$\mathrm{RecV@R}_\gamma(X, Y) = \sup_{\lambda \in [0,1]} \mathsf{V@R}_{\gamma(\lambda)}(X + (1-\lambda)Y).$$

The **solvency condition**

$$\mathrm{RecV@R}_\gamma(E_1, L_1) \leq 0$$

is **equivalent** to requiring that for **<u>all</u> recovery fractions** $\lambda \in [0,1]$ the recovery probabilities satisfy

$$P(A_1 < \lambda L_1) \leq \gamma(\lambda) \quad \Longleftrightarrow \quad P(A_1 \geq \lambda L_1) \geq 1 - \gamma(\lambda).$$

Leibniz Universität Hannover

Recovery of Claims – Stylized ALM-Model (3)

- We return to the simple example introduced above with

$$E_1^k(\omega) = \begin{cases} 100 - k & \text{if } \omega = g, \\ k - 100 & \text{if } \omega = b. \end{cases} \quad \forall\, k \in [0, 100], \qquad L_1(\omega) = \begin{cases} 1 & \text{if } \omega = g, \\ 100 & \text{if } \omega = b. \end{cases}$$

- For a **probability level** $\beta \in (0, \alpha/2)$ and a **desired recovery fraction** $r \in (0, 1)$, we set

$$\gamma(\lambda) = \begin{cases} \beta & \text{if } \lambda \in [0, r), \\ \alpha & \text{if } \lambda \in [r, 1]. \end{cases}$$

- This implies that

$$\mathrm{RecV@R}_\gamma(E_1^k, L_1) \le 0 \iff k \ge 100r,$$

  i.e., the maximal shareholder value is attained for $k = 100r$.
- Liabilities in state b are equal to 100. **This implies that the recovery fraction in state b is equal to r.**

Recovery Average Value at Risk

- The **recovery average value at risk** dominates the RecV@R and is cash invariant in its first component, monotone, convex, subadditive, positively homogeneous, star shaped in its first component, and normalized.

Definition

The **Recovery Average Value at Risk** $\mathrm{RecAV@R}_\gamma : L^1 \times L^1 \to \mathbb{R} \cup \{\infty\}$ with increasing level function $\gamma : [0,1] \to [0,1)$ is defined by

$$\mathrm{RecAV@R}_\gamma(X,Y) := \sup_{\lambda \in [0,1]} \mathrm{AV@R}_{\gamma(\lambda)}(X + (1-\lambda)Y).$$

Portfolio Optimization

- **Efficient frontier**
  - We are interested in optimal combinations of return and downside risk – the *efficient frontier* – but with risk measured by $\mathrm{RecAV@R}$.
  - This problem can equivalently be stated as the minimization of risk for a given expected return.

Problem

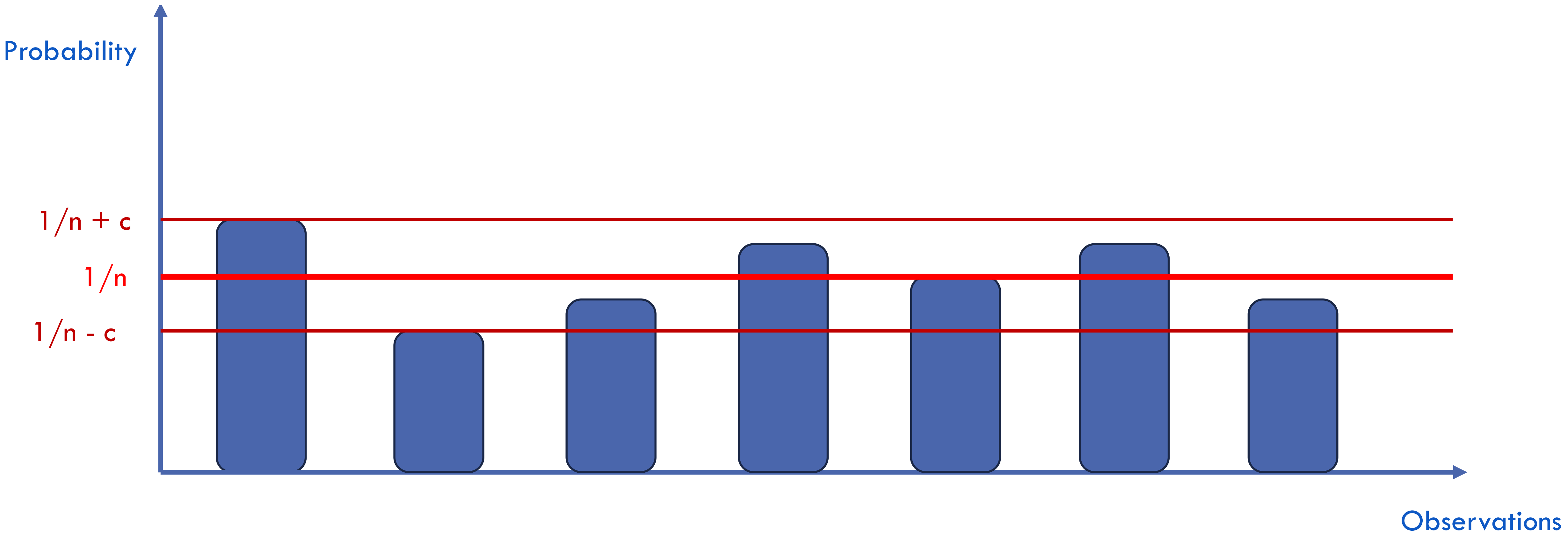
- For a given level function $\gamma : [0,1] \to [0,1]$ and for given $a \in \mathbb{R}$, we consider the following problem:

$$\min_{x \in \Delta^K} \mathrm{RecAV@R}_\gamma(E_1(\boldsymbol{x}), L_1)$$
$$\text{s.t.} \quad \mathbb{E}(A_1(\boldsymbol{x})) \geq a.$$

- Recovery risk measures may successfully be applied to portfolio optimization in practice.
  - For $\mathrm{RecAV@R}$ with suitable recovery functions the characterization of the efficient frontier may, on the basis of a suitable minimax theorem, be reduced to the minimization of a linear function on a convex polyhedron.

Box Uncertainty

In the presence of Knightian uncertainty, we assume that the statistical probability measure is not known. Instead we take a worst-case approach.



Mixture and Box Uncertainty

In the presence of Knightian uncertainty, we assume that the statistical probability measure is not known. Instead we take a worst-case approach.

Techniques for AV@R discussed in Zhu & Fukushima (2009) can be extended to RecAV@R.

- We prove suitable minimax theorems in both cases.
- In both cases, mixture and box uncertainty, one finally arrives at tractable linear programs of the approximations via Monte Carlo simulations.
- Details and numerical experiments are worked out in our paper.

Responsible Portfolio Optimization

- Growing concern about sustainability, due to reasons such as climate change or recent wars
- Include Environment, Social and Governance (ESG) criteria as a third dimension together with risk and return
    - resource use, emissions and innovations
    - workforce, human rights, community and product responsibility
    - management, sharegholders, corporate social responsibility
- In addition to the decision vector $\mathbf{x}$ of portfolio weights and the vector $\mathbf{y}$ of returns, include the ESG sore with the vector $\boldsymbol{\theta} \in [0, 100]^n$.
- Same minimization problem as before but add an ESG constraint with θ_{min} as the minimal ESG portfolio score

$$\sum_{i=1}^{n} x_i \theta_i \geq \theta_{\text{min}}$$

Conclusion

1. Recovery risk measures successfully control recovery.
2. They can successfully be applied to solvency regulation, performance-based management, and portfolio optimization.
3. Future research needs to study their implementation and simulation in complex ALM-models.

Leibniz Universität Hannover

Thank you for your attention!

- Anna Eggert (2023): 'Responsible Portfolio Optimization under AVaR Constraints', Bachelor thesis
- Cosimo Munari, Stefan Weber, & Lutz Wilhelmy (2023): 'Capital Requirements and Claims Recovery: A New Perspective on Solvency Regulation', *Journal of Risk and Insurance*, **90**(2), 329–380.
- Cosimo Munari, Justin Plückebaum & Stefan Weber (2023): 'Robust Portfolio Selection Under Recovery Average Value at Risk', submitted.
- Zhu, S., & Fukushima, M. (2009): 'Worst-case conditional value-at-risk with application to robust portfolio management', *Operations research*, **57**(5), 1155-1168.