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Least-Squares Monte Carlo Methods for Proxy Modeling of Life Insurance Companies

Ralf Korn (TU Kaiserslautern, Fraunhofer ITWM)

Based on:

A.S. Krah, Z. Nikoliç, R. Korn (2018) *A least-squares Monte Carlo framework in proxy modeling of life insurance companies*, Risks 6(2), doi:10.3390/risks6010001

D. Bauer, H. Ha (2018) *A least-squares Monte Carlo approach to the calculation of capital requirements*. Working paper.

See also

A.S. Krah, Z. Nikoliç, R. Korn (2019) *Machine learning in least-squares Monte Carlo proxy modeling of life insurance companies*, working paper.

More references:

later

A motivating problem: Solvency II requirements

European Council Directive 2009/138/EC

Article 122:

“Where practicable, insurance and reinsurance undertakings shall derive the solvency capital requirement directly from the **probability distribution forecast** generated by the internal model of those undertakings, using the Value-at-Risk measure set out in Article 101(3).”

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- **Least-Squares Monte Carlo approach**

1 Least Squares Monte Carlo: Basic idea

A simpler task: Derive the distribution of an option price $g(X(t))$ at time t with a payment of $H = h(X(T))$ at time T

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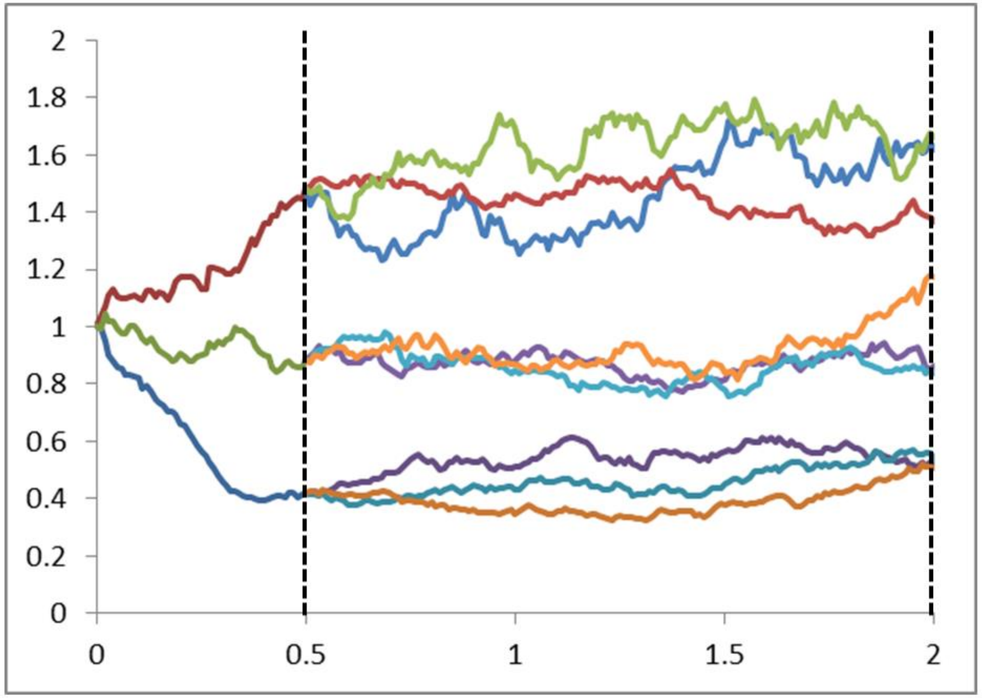

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  - Derive a **regression representation** r(X(t)) for g(X(t)) on the basis of h(X(T))
  - Simulate a huge number of **representative values X(t)** to obtain an **approximation of the distribution function** for **g(X(t))** with the help of **r(X(t))**

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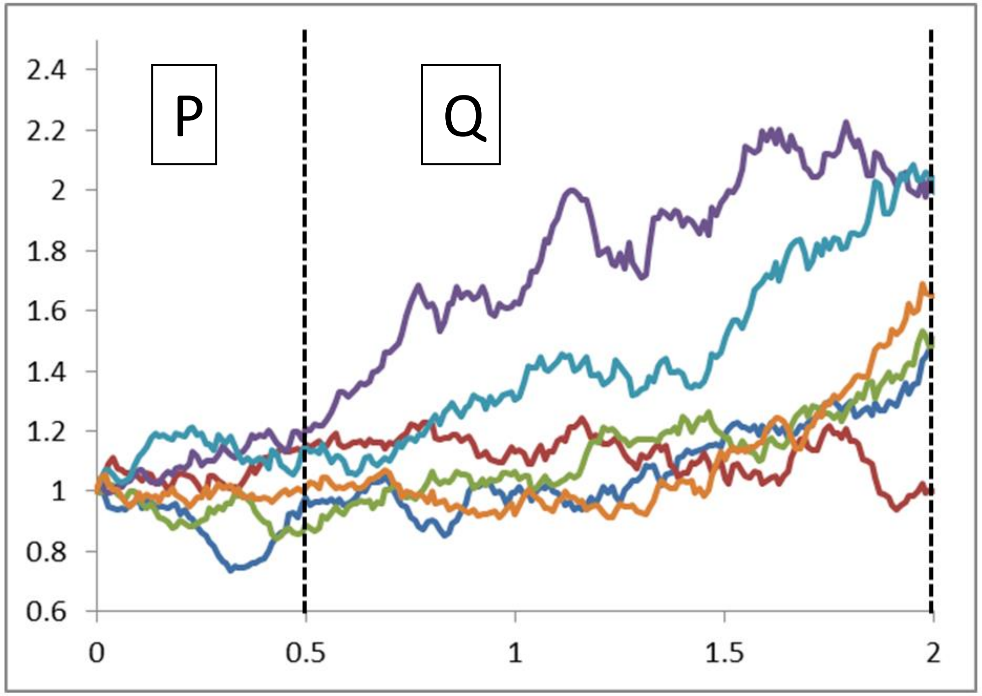
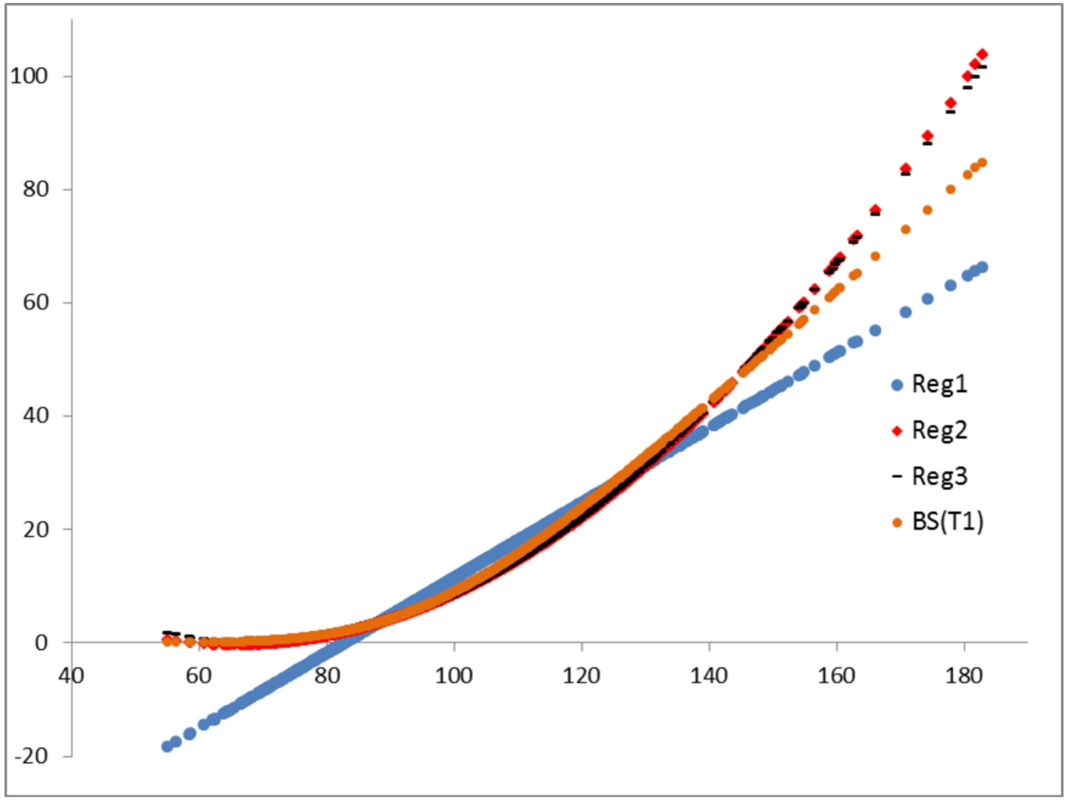


1 Least Squares Monte Carlo: A simple option pricing example

European call option: Derive the distribution of a European call option price g(S(1)) at time 1 with a payment of $H = (S(2)-K)^+$ at time 2 by using a linear, a quadratic and a cubic regression function (still linear in the parameters!)



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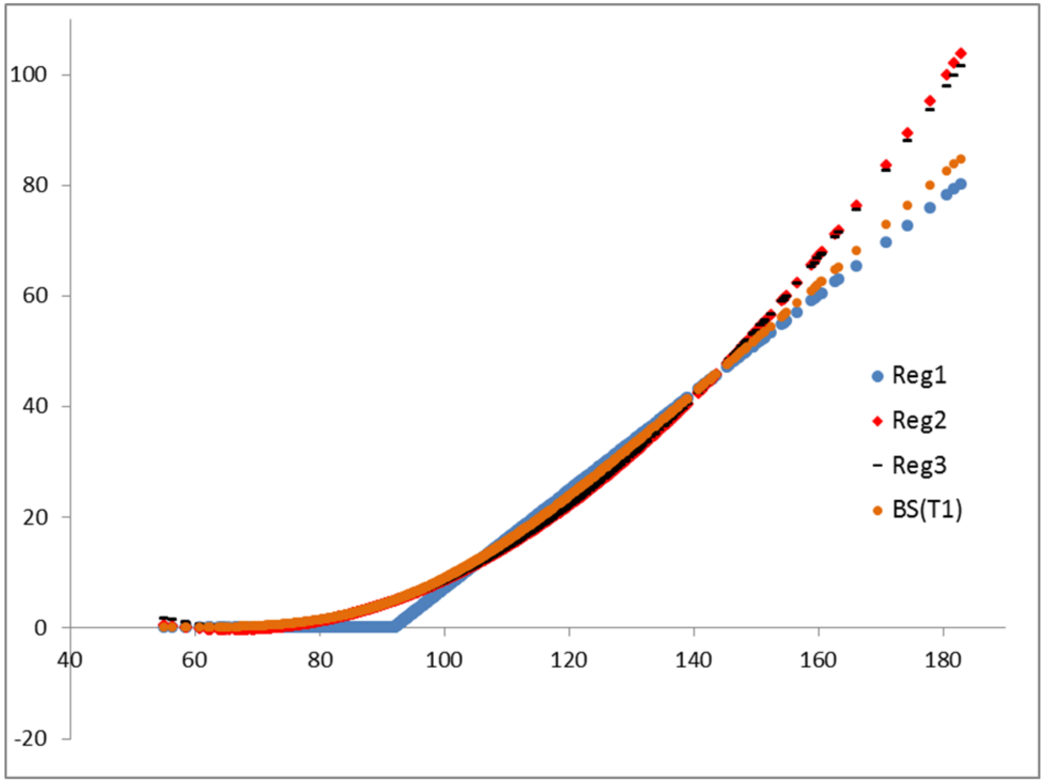
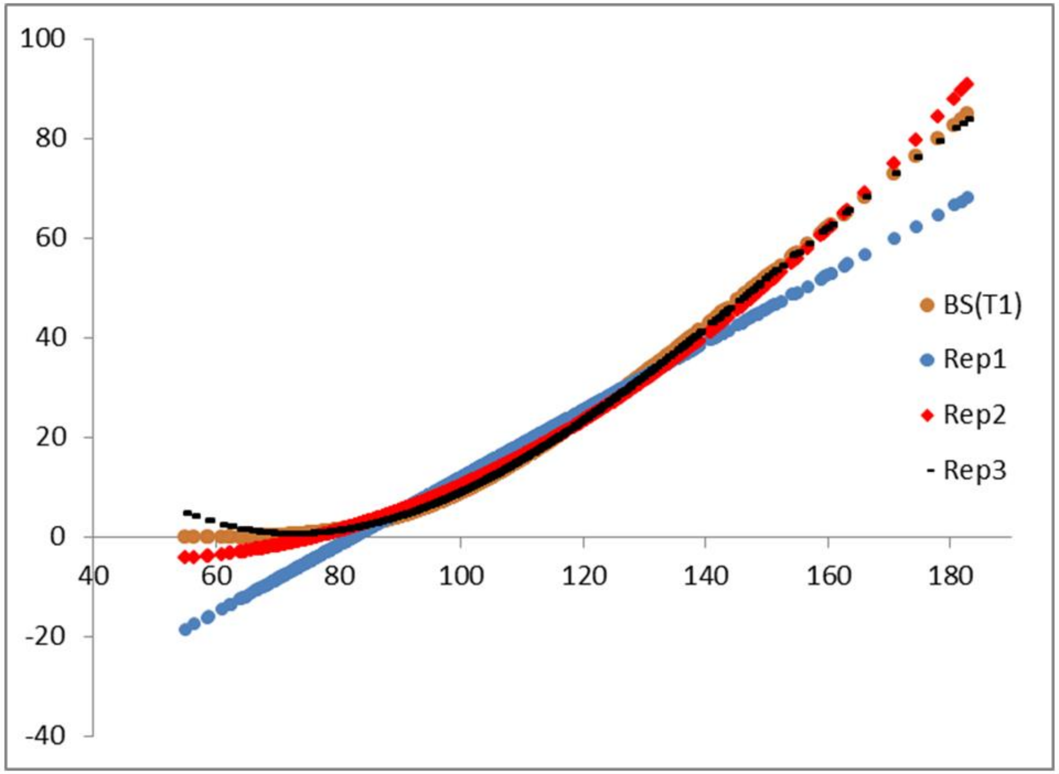


1 Least Squares Monte Carlo: A simple option pricing example

Replication approach: Derive the distrib. of a Eur. call price g(S(1)) at time 1 with a payment of $H = (S(2)-K)^+$ at time 2, use a lin., a quadr. and a cubic regression function for H (!!!) at t =2 and then calculate its price at t=1=> slightly better than LSMC



1 Least Squares Monte Carlo: Main tasks/problems

Theoretical justification:

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Practical implementation:

- Choice of the regression function
- Number of simulation runs for calibration
- How to choose the fitting values?
- How to choose the calibration values?
- How to judge the performance of the model?

1 Least Squares Monte Carlo: Convergence results

Theoretical justification: Convergence results => two convergence issues

Theorem:

Let $F(X) = E(Y|X)$ be a functional of X that is in L^2. Consider a set of K linearly independent basis functions $e_k(x)$ with $e_0(x)=1$, the projection

(1) $$\hat{F}^{(K)}(X)=\sum_{k=0}^{K-1}\beta_k e_k(X)$$

of $F(X)$ on the basis functions and

(2) $$\hat{F}^{(K,N)}(X)=\sum_{k=0}^{K-1}\hat{\beta}_k^{(N)} e_k(X)$$

its approxim. with the LS-estimators of the coefficients based on N realizations of Y.

a) If the family of basis functions is complete in $L^2\left(IR^d, B^d, P\right)$ then we have

(3) $$\hat{F}^{(K)}(X)\xrightarrow{K\to\infty}F(X) \quad \text{in } L^2\left(IR^d, B^d, P\right)$$

b) $$\hat{F}^{(K,N)}(X)\xrightarrow{N\to\infty}F^{(K)}(X) \quad \text{a.s.}$$

2 Least Squares Monte Carlo: Aspects of application

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- A **simulation concept** how to cover the relevant (!) values of the risk factors (i.e. the ones that are relevant for the loss distribution and for the distribution at the calculation time)

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- A **simulation concept** how to cover the relevant (!) values of the risk factors (i.e. the ones that are relevant for the loss distribution and for the distribution at the calculation time)
- A decision on the **method to actually determine the loss distribution** and in particular the relevant high/low quantiles for the **Solvency Capital Requirements**

2 Least Squares Monte Carlo: Aspects of application – 2

The necessary key steps/decisions/ingredients of the LSMC approach on the way to a reliable proxy modelling for a life insurance company:

- a detailed description of the simulation setting and the required task
- a concept for a calibration procedure for the proxy function
- a validation procedure for the obtained proxy function
- the actual application of the LSMC model to forecast the full loss distribution

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First part:

Given by the CFP method and by the SCR requirement/definition, in particular by specifying the risk factors $X= (X_1,\dots,X_d)$ the insurer is exposed to in the next year.

A realization of X under the subjective measure P is called an **outer scenario** (i.e. one possibility how the world will evolve during that year.

3 LSMC-Proxy Modelling: Simulation setting and the task

The task(s):

- Calculate the (full) **loss distribution** of a life insurance company (over a given time horizon) at the end of the year
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The simulation setting:

- Simulate realizations of **risk factors** X at time 1 under P
- For each realization of the risk factors derive the (discounted) available capital at time 1:

$$AC(X) = E_Q\left(\sum_{t=1}^{T} B_t^{-1} Z_t \,|X\right) =: E_Q\left(\sum_{t=1}^{T} z_t\left(\phi_t(X)\right)|X\right)$$

where Z_t denotes the net profit at time t and let T mark the projection end. Note that we simulate now, i.e. we use the CFP method available now!

3 LSMC-Proxy Modelling: Simulation setting and the task – 2

An example

Table 1. Risk factors in the CFP model.

Component	Risk Factor Description
X_1	Risk-free interest rates movement
X_2	Change in interest rate volatility
X_3	Change in equity volatility
X_4	Shock on volatility adjustment (if used by the company)
X_5	Credit default
X_6	Credit spread widening
X_7	Currency exchange rate risk
X_8	Shock on equity market value
X_9	Shock on property market value
X_{10}	Lapse stress on best estimate assumptions
X_{11}	Mortality catastrophe stress with a one-off increase in mortality
X_{12}	Mortality trend volatility stress
X_{13}	Mortality level stress on best estimate assumptions
X_{14}	Longevity trend volatility stress on best estimate assumptions
X_{15}	Longevity level stress on best estimate assumptions
X_{16}	Morbidity stress on best estimate assumptions
X_{17}	Expenses stress on best estimate assumptions

Capital market shocks (X_1–X_9)

Actuarial risks (X_{10}–X_{17})

3 LSMC-Proxy Modelling: Simulation of the outer and inner scenarios

Generate the outer scenarios (the **fitting points) under P, i.e.**

- have a stochastic model for each risk X_i and simulate realizations $X^{(k)}$ of X,
- use a large number of outer scenarios (or have a strategy how to fill the range of the risks)

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Generate the corresponding inner scenarios (the **fitting values) under Q, i.e.**

- use an economic scenario generator (ESC) for generating very few (typically 1 or 2) market consistent scenarios $\phi^{(k,j)}(X^{(k)})$ for each outer scenario
- derive the fitting values $Y^{(k)}$ via

$$Y^{(k)} = \frac{1}{a}\sum_{j=1}^{a} Y^{(k,j)} = \frac{1}{a}\sum_{j=1}^{a}\sum_{t=1}^{T} z_t\left(\phi^{(k,j)}(X^{(k)})\right)$$

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Note: Pairs $(X^{(k)},Y^{(k)})$ for setting up a regression function are now available

Main question: How to choose the regression function?

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Suggestion:

Use monomials of the type

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Find the least-squares optimal coefficients based on the N fitting points and fitting values to obtain the proxy function by solving

$$\hat{\beta}^{(N)} = \arg\min_{\beta \in IR^K} \left\{ \sum_{i=1}^{N} \left(Y^{(i)} - \sum_{k=0}^{K-1} \beta_k e_k \left(X^{(i)} \right) \right)^2 \right\}$$

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Example of choice by marginalization:

$x_1^2 x_2$ can only be among the candidates if $x_1^2, x_1 x_2, x_1, x_2$ are already choosen

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Some out of-sample-tests are described in Krah et al. (2018)

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- Accuracy check good, total computation time < 6 hours on a cluster , nested simulation is estimated to last at least 25 weeks

More details in Krah et al. (2018)

Thank you for your attention !

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