Some Insurance Valuation and Design Problems with Aggregate Risk

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OUTLINE

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OVERVIEW

Standard insurance valuation/design problems

- Pooling homogeneous, (conditionally) independent risks
- Representative agent/policyholder
- If portfolio is large, only aggregate risk matters

In practice, however...

- Aggregate risk can arise endogenously (e.g., policyholder behavior)
- Valuation and contract design should internalize aggregate risk

Some interesting problems

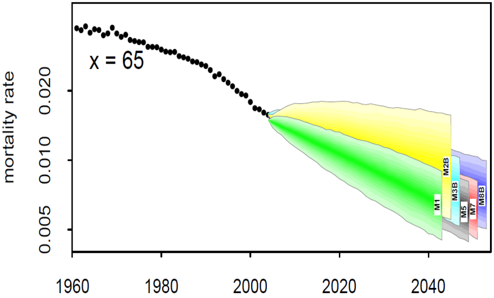
- Optionality in long term insurance contracts
    - Ex-ante i.i.d. risks give rise to endogenous aggregate risk
- P&C examples
    - Conditionally i.i.d. risks and coverage for high layers of exposure
    - Multi-year agricultural insurance in supply chain risk management

OUTLINE

OPTIONALITY IN LIFE INSURANCE

Long term insurance contracts

- Longevity/mortality risk assessment: is it enough?
- Are financial and demographic risk factors uncorrelated?
- Asset Management Charges (AMCs) vs. level premiums
- Role of contract design and policyholder behavior
- Endogenous dependence and aggregate risk via optionality



LifeMetrics mortality fan charts. Source: Dowd et al. (2008).

Policyholder behavior

- **'Rational'** exercise of options
- **Testing** for dynamics adverse selection
- Making sense of actuarial approaches: **pricing basis** & and **lapse/surrender basis**

SETUP

Longevity risk

- Aggregate changes in survival probabilities
- Both aggregate and idiosyncratic risk relevant in the presence of optionality

Reference setup: conditionally Poisson / Cox setting (more generally, see Tappe and Weber, 2014)

- At contract inception (time 0), portfolio of insureds with **death** times $\tau^1, \ldots, \tau^n$
- Each τ^i has force of mortality $\mu^i(t)$
- Possible representations: $\mu^i(t) = X(t) + Y^i(t)$ or $\mu^i(t) = X(t)\, Y^i(t)$

Portfolio vs. population

- **Surrender/lapse** time θ^i
- **Exit** from the portfolio at stopping time $\sigma^i := \tau^i \wedge \theta^i$

POLICYHOLDER BEHAVIOR

Value of the contract to insured i is

$$v^i(t;\sigma^i,c) = 1_{\sigma^i>t}\mathbb{E}^{\mathbb{Q}^i}\left[\int_t^{\theta^i\wedge T} e^{-\int_t^s (r(u)+\mu^i(u))\mathrm{d}u}\mathrm{d}G^i(s;c)\middle|\mathcal{F}_t^i\right].$$

- $G^i(t;c)$: cumulative gains to the insured from holding the insurance contract, with $c \in \mathcal{C}$ contract configuration (including guarantees)

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Some issues...

- $\mathbb{Q}^i$ private valuation of insured i
- $\mathbb{F}^i := \left(\mathcal{F}_t^i\right)_{t\geq 0}$ (private) information available to insured i
- Endogenous σ^i (optimal stopping problem θ^i)
  - More generally, one should also allow for other dimensions of optionality (fund switches, partial withdrawals, etc.)

Question: how to proxy for v^i across p/h's?

DYNAMIC ADVERSE SELECTION

Individuals **ex-ante** identical

- At contract inception (time 0) policyholders' death times $\tau^1, \dots, \tau^n$ have (say) independent intensities $\mu^1, \dots, \mu^n$ with the same law as process μ
- $(F(t))_{t\geq 0}$ vector of financial risk factors (say) independent of mortality

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Ex-post mortality profile of the portfolio

- Different trajectories $(\mu^i(t, \omega_1), F(t, \omega_1))_{t \geq 0}, \ldots, (\mu^i(t, \omega_k), F(t, \omega_k))_{t \geq 0}$ make staying in the contract more or less valuable for p/h i
- The moneyness of any guarantee/option is at shaped at least by μ^i and $c \in \mathcal{C}$ (contract design channel)
- **Portfolio mortality** (average intensity)

$$\overline{\mu}_p(t) := \frac{\sum_{i=1}^n \mu^i(t) 1_{\sigma^i > t}}{\sum_{i=1}^n 1_{\sigma^i > t}}.$$

- The insurer cannot observe μ^i, but can try to recover the law of $\overline{\mu}_p$ based on $c \in \mathcal{C}$ and relevant (observable) state variables

FRAILTY REPRESENTATION

Change in intensity process

- Think of death times τ (representative member of the population) and $\overline{\tau}_p$ (average portfolio member)
- Dynamic frailty representation: **individual** (on $\{\sigma^i > t\}$) or **average/representative** portfolio member (on $\{\sigma^{(n)} > t\}$)

$$\mu^i(t) = \mu(t)\eta^i(t;c) \qquad \overline{\mu}_p(t) = \mu(t)\overline{\eta}(t;c)$$

  with $(\eta^i(t,c))_{t\geq 0} > 0$ and $(\overline{\eta}(t;c))_{t\geq 0} > 0$ dynamic frailty processes; under suitable assumptions, the Cox setting is preserved (e.g., Biffis, Denuit, Devolder, 2010)
- Think of change in intensity as captured by a suitable change of probability measure: likelihood ratio driven by dynamic frailty process

PRICING

Insurer's view

- Baseline reference probability measures $\mathbb{Q}_F$ (financial factors) and $\mathbb{P}_M$ (population mortality)
- Pricing with $\mathbb{Q} := \mathbb{Q}_F \otimes \mathbb{P}_M$ (wrong!)

$$V^i(0;\theta^i,c) = V(0;\theta,c) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^{\theta\wedge T} e^{-\int_0^s (r(u)+\mu(u))\mathrm{d}u}\mathrm{d}G(s;c)\right].$$

PRICING

Insurer's view

- Baseline reference probability measures $\mathbb{Q}_F$ (financial factors) and $\mathbb{P}_M$ (population mortality)
- Pricing with $\mathbb{Q}_p$ (reflects portfolio mortality)

$$V_p^i(0;\theta^i,c) = V_p(0;\theta,c) = \mathbb{E}^{\mathbb{Q}_p}\left[\int_0^{\theta\wedge T} e^{-\int_0^s (r(u)+\mu_p(u))\mathrm{d}u}\mathrm{d}G(s;c)\right].$$

- The representative policyholder's death time is τ_p and not τ...

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Implications

- Change in intensity and no factorization in general even if mortality and financial risk factors uncorrelated
- Surrender/lapse basis jointly determined with mortality basis
- Useful framework for contract design: optimize with respect to $c \in \mathcal{C}$
  - ⋆ Determine fair AMCs
  - ⋆ Steer the portfolio toward a target mortality risk profile

EXAMPLES

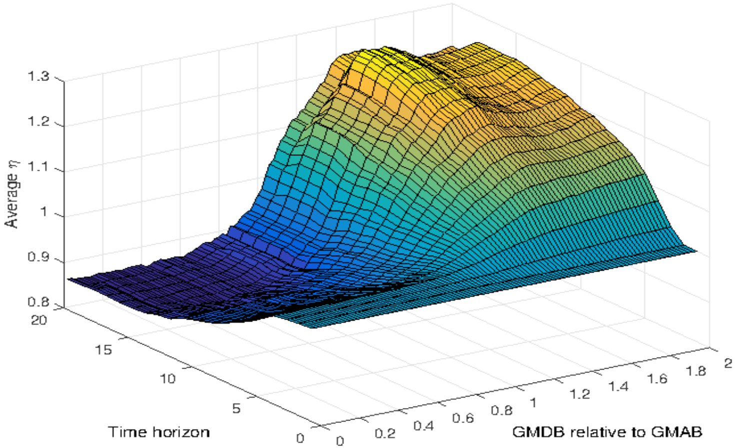
Baseline example

- 20-year VA contract
- 45 male, non smoker
- GMAB (accumulation): 2.5% p.a.
- GMSB (survival): premiums paid with 0% or 2.5% p.a. guarantee; but surrender penalties in the first 5 years of contract
- GMDB (death): varying from zero to $2\times$ GMAB guaranteed rate
- Reference fund: Geometric Brownian Motion, 15% volatility

GMWB (withdrawal) and GMLB (lifetime) also interesting...

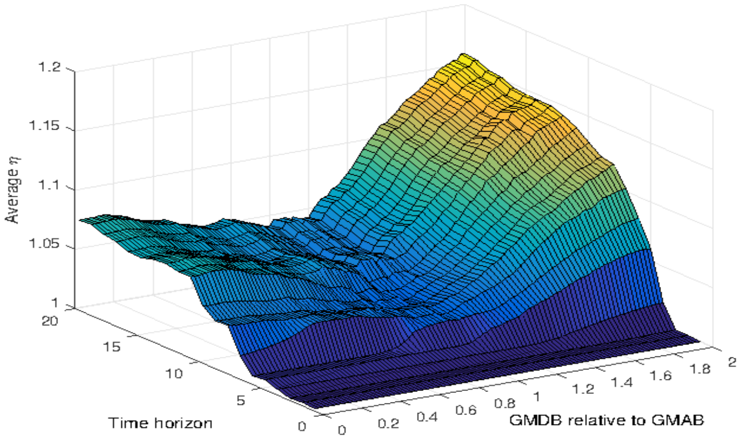
- Wedge between systematic and idiosyncratic risk more important

AVERAGE FRAILTY (GMSB: premium paid)



Source: Benedetti and Biffis (2016).

AVERAGE FRAILTY (GMSB: premium paid rolled over at 2.5% p.a.)



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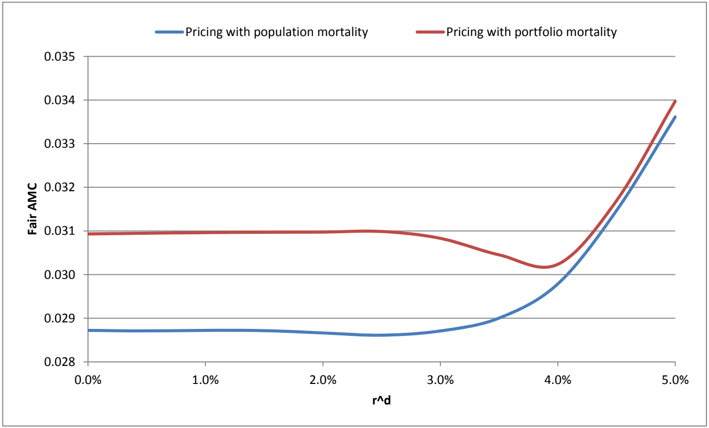
FAIR AMCs (GMSB: initial amount paid into the policy)



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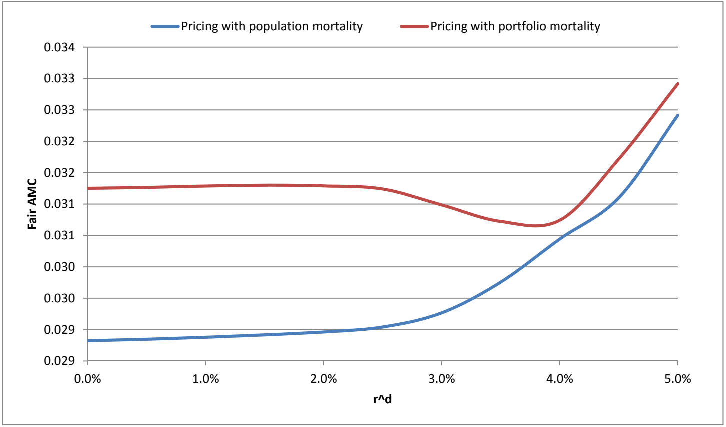

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TESTING FOR DYNAMIC ADVERSE SELECTION

Possible approaches suggested by our framework

- Use frailty process $(\overline{\eta}(t;c))_{t\geq 0}$
- Use 'distance' between $\mu(t)$ and $\overline{\mu}_p(t)$
- Use 'distance' between (conditional) law of τ and $\overline{\tau}_p$

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A class of divergences (e.g., Vonta-Karagrigoriou, 2010)

$$D^{\psi}_{\tau,\overline{\tau}_p}(t) = \int_t^T \psi\left(\frac{\mathrm{d}\mathbb{P}(t<\overline{\tau}_p\leq s|\mathcal{F}_t)}{\mathrm{d}\mathbb{P}(t<\tau\leq s|\,\mathcal{F}_t)}\right)\mathrm{d}\mathbb{P}(t<\tau\leq s|\,\mathcal{F}_t),$$

with $\psi\in C^2(\mathbb{R}_+;\mathbb{R})$, $\psi(1)=0$

- Examples: α-divergences (Csiszàr's family), Kullback-Leibler, Hellinger, etc.
- Different from standard approaches (e.g, Albert et al., 1999; He, 2011)
  - $\texttt{Actual\_deaths}_t/\texttt{Expected\_deaths}_t = \alpha + \beta \times \texttt{Lapse\_ratio}_t + \varepsilon$
  - $\mathbb{P}(\texttt{lapse}_i = 1) = F(a + b\times \texttt{health\_shock}_i)$

SOME RESULTS

β estimates for regressions
$y_{t+1} = \alpha + \beta \times \texttt{lapse\_ratio}_t + \varepsilon_t$.

D/S	$y_{t+1} = \overline{\eta}$ β	p-value	$y_{t+1} = KL(\mu, \mu_p)$ β	p-value
0	-1.62*	0.032	0.10*	0.027
0.1	-1.94*	0.008	0.11*	0.006
0.3	-2.17*	0.009	0.08	0.055
0.5	-24.02	0.005	1.45*	0.006
0.7	-2.52*	0.020	0.21*	0.004
0.9	-0.71	0.146	0.14*	0.000
1.1	-0.43	0.246	0.12*	0.001
1.3	-0.26	0.355	0.12*	0.002
1.5	-0.13	0.434	0.13*	0.002
1.7	-0.13	0.442	0.12*	0.002
1.9	-0.28	0.380	0.13*	0.002

Source: Benedetti and Biffis (2016).

β estimates for regressions
$y_{t+1} = \alpha + \beta \times \texttt{lapse\_ratio}_t + \gamma \times t + \varepsilon_t$.

$y_{t+1} = \overline{\eta}$ β	p-value	$y_{t+1} = KL(\mu, \mu_p)$ β	p-value
-1.83*	0.043	0.11*	0.029
-2.28*	0.010	0.14*	0.004
-2.18*	0.022	0.11*	0.039
-27.75*	0.006	1.58*	0.009
-2.95*	0.018	0.21*	0.010
-1.04	0.114	0.13*	0.004
-0.82	0.167	0.12*	0.011
-0.70	0.241	0.10*	0.038
-0.54	0.324	0.13*	0.022
-0.62	0.326	0.15*	0.011
-0.68	0.335	0.14*	0.030

Source: Benedetti and Biffis (2016).

- Simulated environment for 2500 traditional contracts issued to male non-smokers aged 50.
- Maturity $T = 20$ years, decreasing surrender penalties during the first 3 years of contract. Death (D) and survival (S) benefits.
- Use average frailty $\overline{\eta} = \overline{\mu}_p/\mu$ as proxy for actual/expected deaths.

OUTLINE

RISK SHARING AND LIMITED LIABILITY

A risk sharing problem (Arrow/Raviv) with limited liability

- One-period model with a continuum of insurees modeled as the measure space $(M, \mathcal{M}, \mu)$ of the unit interval $M = [0, 1]$, with $\mu(M) = 1$.
- Insurer maximizes function V over indemnities (I_i), and risky asset allocation (α)

$$V(\alpha, (I_i)) = \max\left\{ \left(A + \int_0^1 \pi_i \mu(di)\right)(1 + \alpha R) - \int_0^1 I_i(X_i)\mu(di), 0 \right\}$$

where $I_i(X_i)$ is indemnity for p/h i's loss X_i financed by insurance premium $\pi_i \geq 0$

  - Can optimize relative to initial capital A
  - Can add regulatory constraints

- Each insuree satisfies the participation constraint

$$E\left[u_i(w_i - \pi_i - X_i + I_i(X_i)1_{D=0} + \gamma I_i(X_i)1_{D=1})\right] \geq \underline{u}_i,$$

with $\{D = 1\}$ default event, $\gamma \in [0, 1]$ recovery rate

AGGREGATION

Assume $X_i = Y_i + Z$ for all $i \in [0,1]$

- (Y_i) essentially uncorrelated (and i.d. for simplicity here), $(Y_i), Z \in L^2$
- Use Sun (2006)'s Exact Law of Large Numbers.

Some special cases

- Idiosyncratic risk only $(Z = 0)$

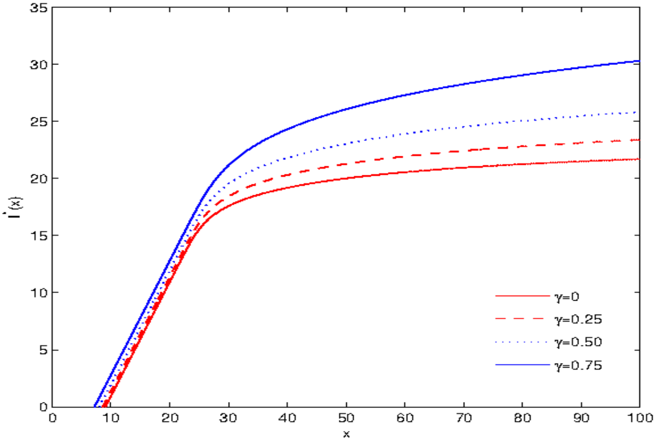
$$\int_0^1 I(X_i)\mu(di) = \int_0^1 E[I(X_i)]\mu(di) = E[I(X_i)] = E[I(X)] \text{ a.s.}$$

- Systematic risk only $(Y_i = 0)$: some examples to follow

$$\int_0^1 I(X_i)\mu(di) = \int_0^1 E[I(X_i)|Z]\mu(di) = E[I(X_i)|Z] \; ..$$

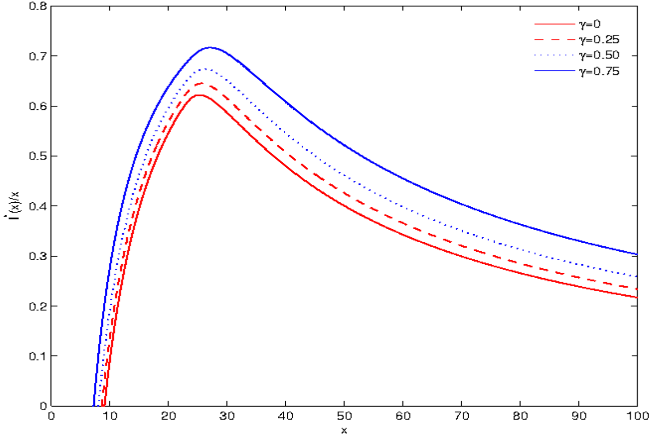
- Good model lies somewhere in the middle

OPTIMAL INDEMNITY SCHEDULE



Source: Biffis and Millossovich (2013).

OPTIMAL RETENTION LEVELS



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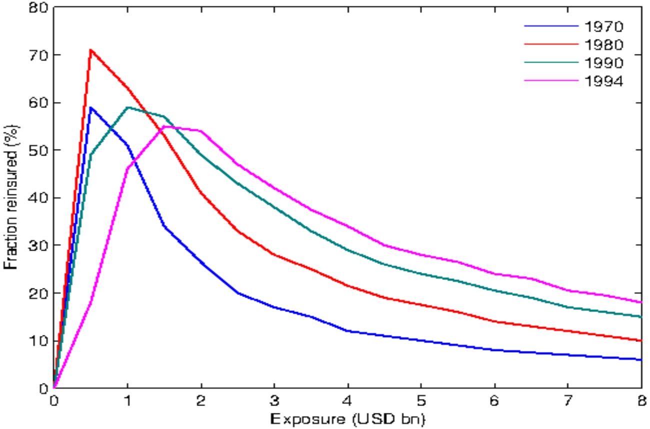
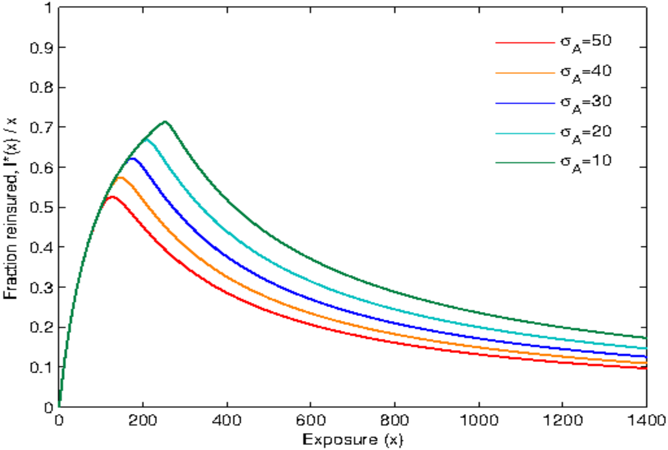
RETENTION/COVERAGE OF HIGH LAYERS OF EXPOSURE: EVIDENCE



Average retention levels in US P&C, evidence from reinsurance purchases. Source: Guy Carpenter (e.g., Froot 1997,2001).

REINSURANCE PURCHASES



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SUPPLY CHAIN RISK MANAGEMENT

General questions

- How to unlock value in supply chains via risk sharing arrangements?
- How to build inclusive and resilient local-to-global supply chains?

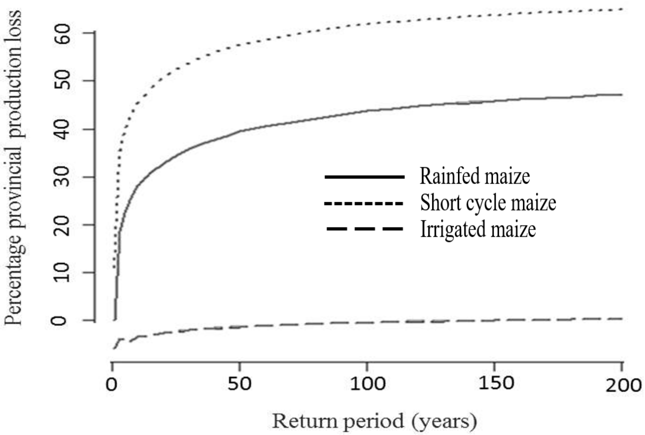
Agricultural insurance example (World Food Program)

- Farmers organizations as aggregators of small farmholders
- Banks as providers of credit (better inputs and technology)
- Agro-dealers as off-takers
- (Re)insurers cover extreme crop yield losses

Challenges (World Food Program)

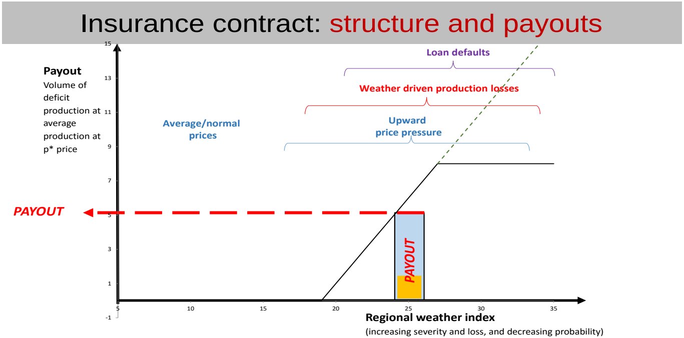
- How to incentivize farmers to switch to more resilient production technologies?
- Technology takes time to demonstrate its value (several harvesting seasons)
- At odds with short term contracts offered by (re)insurers

PRODUCTION TECHNOLOGIES



Source: Biffis and Chavez (2016).

MULTI-YEAR PROGRAMS

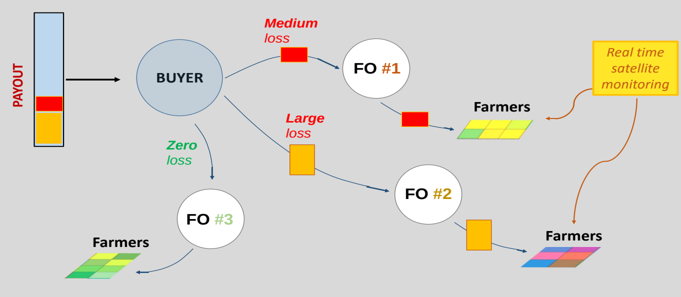


Source: WINnERS project, Biffis and Chavez (2016).

- Uncertainty in medium-to-long-term climate projections is source of aggregate risk
- Explicitly allow for random fraction (Q) of farmholders affected by crop yield losses
- Optimal contract $I^*(X, Q)$ entails contingent attachment/detachment points (Biffis and Louaas, 2016)

MULTI-YEAR PROGRAMS

Insurance contract: indirect insurance for farmer



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CONCLUSION

Standard valuation/risk sharing models useful

- Risk pooling (predictability, vanishing cost of capital)
- Representative policyholder approach

Allowing explicitly for aggregate risk can be more useful

- From idiosyncratic risk to systematic risk via optionality
- Systematic risk, aggregate risk, and counterparty risk
- New avenues for risk sharing via complete contracts

Technical caveats

- Some interesting challenges: incomplete market valuation methods and feedback effects, existence and uniqueness of solutions in risk sharing problems, etc.

THANK YOU