Modeling Traffic Accidents Caused by Random Misperception

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***Abstract*— Understanding the formation of accidents is of major importance to the automotive industry, its related businesses and policymakers. This is not a trivial task considering the current stream of innovations driven by the development of autonomous vehicles. Historical accident data are inadequate for gauging the safety of future traffic systems. To cope with this challenge, we propose a microscopic traffic model that introduces small errors due to random misperception as an omnipresent cause for accidents – an issue affecting both human drivers and control systems of autonomous vehicles. We model errors dynamically by stochastic processes and investigate their impact on the safety and the efficiency of traffic systems by Monte Carlo simulations. We focus on two case studies: a simple one-lane road segment and a t-junction with turning vehicles.**

I. Introduction

Classically, the problem of determining the probability of traffic accidents has been of statistical nature: On the basis of empirical data, the probabilities of accidents are estimated by the corresponding relative frequencies. However, the automotive industry is undergoing a massive disruption with the appearance of autonomously driving vehicles (cf., e.g., [1]). Traffic, as we know it, will change; in particular, in terms of efficiency and safety – but little to no data is available, yet! As long as operations are protected from large-scale cyber attacks, existing studies (e.g., [2]) indicate that the number of accidents will significantly be reduced when vehicles are controlled by computers. To overcome the lack of empirical accident data for future transportation systems, we propose a simulation based approach that yields insight into the occurrence of accidents and their effects on traffic flow.

Both human drivers and control systems of autonomous vehicles need to process large amounts of information about their environment. In most theoretical traffic models, decisions are based on exact information – in reality, errors may occur when positions and velocities of other vehicles are determined. The size of these errors depends on external conditions (e.g., weather) and on the driving style of a human operator or control algorithm. Another factor are potential malfunctions of systems. This paper presents a stylized model for potential errors and investigates the impact on accidents and traffic flow. The key idea is to focus on *random misperception* as an omnipresent cause for accidents. Particular emphasis is put on the interplay between safety gaps and margins of perceptional errors. On a methodological level, the model facilitates an understanding of risks that are associated to beneficial future developments. Ultimately, our approach and techniques may form a basis for management decisions on the design of safety measures for autonomous driving systems.

We choose the Intelligent Driver Model (IDM) (cf. [3]) as the underlying model for describing the movement of vehicles on lanes. Additionally, we incorporate adjustments allowing for driving errors that may lead to accidents. The IDM sets the acceleration of a vehicle based on the distance to its preceding vehicle and the difference of their velocities, i.e., the approaching rate. As originally proposed, this model is accident-free since the maximal deceleration is unbounded and, consequently, vehicles may execute unrealistic emergency braking maneuvers when they encounter dangerous situations. We modify the IDM at this point and also include the possibility of random misperception. The consequences of these changes are investigated in the context of two scenarios.

In Scenario A ("One-Lane Road Segment"), traffic is considered on a segment of a one-lane road on which vehicles drive in a consecutive order. We include two adjustments to the IDM: First, instead of assuming that the input variables (distance and approaching rate) are known with absolute precision, we include stochastic deviations in order to model random misperception; both distance and approaching rate may be over- or underestimated. Second, we limit the deceleration when braking, i.e., negative acceleration is bounded from below. With these two components, the model admits accidents. Whenever an accident occurs, the road is blocked and a traffic jam emerges. We assume that the collided vehicles are removed from the road after a random time and then traffic resumes. In this scenario, we focus on the occurrence of rear-end collisions. By means of Monte Carlo simulations, we study the tradeoff between safety and efficiency in terms of the number of accidents and traffic flow.

Scenario B ("Left-Turning on T-Junction") is an extension that builds on the first scenario. We consider a more complex element of a road system: a simplified t-junction. We capture this by considering two one-lane road segments which intersect. On each lane, the movement of vehicles is modeled as before; moreover, a conflict detection and reaction is implemented for vehicles which turn left. Turning vehicles extrapolate trajectories of conflicting vehicles on the basis of several observations. If the analysis of these trajectories suggests a collision, the turning vehicle will decelerate to allow conflicting vehicles to pass. We implement random misperception in the conflict detection and reaction behavior and use Monte Carlo simulations to analyze traffic at t-junctions, focusing again on the number of accidents and traffic flow.

The paper is organized as follows: Section II reviews mathematical prerequisites. Section III presents the traffic model. Section IV describes numerical case studies and analyzes the tradeoff between safety and efficiency. Section V concludes and discusses further research.

Literature. The analysis of the tradeoff between safety and efficiency of autonomous vehicles is a novel area of research. Most closely related to our approach is [4] who analyze emergency braking scenarios on the basis of a deterministic IDM. This paper also observes the necessity to bound deceleration in order to observe accidents. The focus lies on the impact of inter-vehicle communication on safety, and the computation terminates whenever an accident occurs. Similar ideas can also be found in [5]. Stochastic extensions of the IDM are introduced in [6]. Random misperceptions provide a rationale for the empirical behavior of human drivers that is characterized by fluctuating accelerations. Such an approach is also studied in [7], [8], and [9]. These papers add white noise to the acceleration terms of car-following models. Accidents are, however, not investigated.

II. Mathematical Foundations

In this section, we review random ordinary differential equations and Ornstein-Uhlenbeck processes. These are ingredients to our traffic model with random misperception.

A. Random Ordinary Differential Equations

The classical IDM is described by ordinary differential equations (ODEs). Random misperception leads to a stochastic analogue of the equations, *random ordinary differential equations* (RODEs). We briefly describe RODEs and how to solve them; a comprehensive presentation of RODEs can be found in [10].

Definition 1 (Random Ordinary Differential Equation). *Let $(\varepsilon_t)_{t\geq 0}$ be a stochastic process on some probability space $(\Omega, \mathcal{F}, P)$ with values in $\mathbb{R}^m$ and continuous paths. Suppose that $f\colon \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^d$ is continuous. A* random ordinary differential equation *in $\mathbb{R}^d$ for some function $y\colon [0,\infty) \to \mathbb{R}^d$ is given by*

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y, \varepsilon_t).$$

For each scenario $\omega \in \Omega$, a RODE defines a non-autonomous ordinary differential equation via

$$\frac{\mathrm{d}y}{\mathrm{d}t} = F_\omega(t,y) := f(y, \varepsilon_t(\omega)).$$

Given $y(0) = y_0 \in \mathbb{R}$, this is a standard initial value problem and classical ODE-theory (e.g., Theorem of Picard-Lindelöf) applies when characterizing existence and uniqueness of solutions. Pathwise RODEs are ODEs which also allows to use standard numerical methods for ODEs in order to solve RODEs. This approach can be applied whenever sufficiently many realizations of the paths of the underlying stochastic process (ε_t) are available.

B. Ornstein-Uhlenbeck Process

In the extended IDM we model misperceptions as random deviations from the true values. This can be captured by a mean-reverting process in continuous time. A well-known Gaussian process of this type is the Ornstein-Uhlenbeck process.

Definition 2 (Ornstein-Uhlenbeck Process). *Let $\beta \in \mathbb{R}$ and $\alpha, \sigma > 0$. A stochastic process $(\varepsilon_t)_{t\geq 0}$ is called an* Ornstein-Uhlenbeck process, *if $\varepsilon_0 = a \in \mathbb{R}$ and $(\varepsilon_t)_{t\geq 0}$ solves the following stochastic differential equation:*

$$d\varepsilon_t \;=\; \alpha(\beta - \varepsilon_t)dt \;+\; \sigma dW_t,$$

where $(W_t)_{t\geq 0}$ denotes a one-dimensional standard Brownian motion.

Following [11], an Ornstein-Uhlenbeck process can iteratively be simulated exactly on an equidistant time grid $0 = t_0 < t_1 < \dots < t_N$ with $t_{i+1} - t_i = \Delta t > 0$ for all $i \in \{0, 1, \dots, N-1\}$ by

$$\varepsilon_{t_{i+1}} = h\varepsilon_{t_i} + \beta\,(1-h) + \sigma\sqrt{\frac{1-h^2}{2\alpha}} Z_{i+1}$$

where $h := e^{-\alpha\Delta t}$ and (Z_i) is a sequence of i.i.d. standard normal random variables. In Fig. 1, we show typical simulated paths of the Ornstein-Uhlenbeck process for different values of σ. The parameter σ is the *volatility* of the process and captures both its tendency to fluctuate as well as the size of the infinitesimal random innovations.

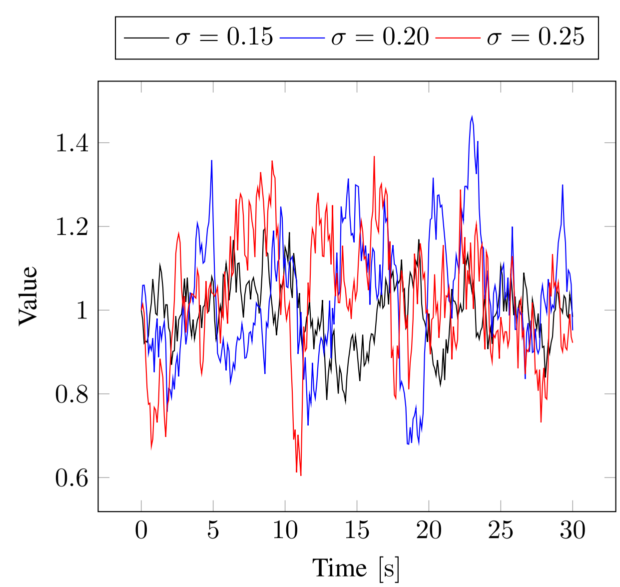

Fig. 1. Simulated paths of an Ornstein-Uhlenbeck process (ε_t) for different values of σ with $\alpha = 1$, $\varepsilon_0 = 1$, $\beta = 1$.

III. The Traffic Model

The traffic models we consider are car-following models: cars react to preceding vehicles in order to maintain a minimum safety distance and to avoid crashes. Among car-following models, the IDM has attracted a lot of attention.

In this section, we extend the IDM and define the novel Intelligent Driver Model with Random Misperception: first by bounding the maximal deceleration, and second by introducing random misperception. As a consequence, accidents may occur.

A. Movement of Vehicles

We denote by $\mathcal{M} := \{1, 2, \dots\}$ the collection of vehicles. Each vehicle $i \in \mathcal{M}$ drives on a one-lane road modeled by a one-dimensional line $[0, L]$ of length $L > 0$; it enters the road at a time $t_0^i \geq 0$. The time sequence is increasing, i.e., $t_0^1 < t_0^2 < \dots$.

The velocity of each vehicle is determined according to the *Intelligent Driver Model with Random Misperception* (IDMrm): Let $(\varepsilon_t^{i,1})_{t\geq 0}, (\varepsilon_t^{i,2})_{t\geq 0}, (\varepsilon_t^{i,3})_{t\geq 0},\ i \in \mathcal{M}$, be independent stochastic processes with continuous paths. The IDMrm sets the velocity $v^i(t)$ and the position $x^i(t)$ of vehicle $i \in \mathcal{M}$ at time $t \geq 0$ according to the following initial value problem composed of a system of coupled random ordinary differential equations

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} x^i(t) = \max\{v^i(t), 0\}, \\ \frac{\mathrm{d}}{\mathrm{d}t} v^i(t) = \max\left\{ a^i_{\max} \cdot \left(1 - \left(\frac{\varepsilon_t^{i,1} v^i(t)}{v^i_{\mathrm{d}}} \right)^{\delta} \right.\right. \\ \qquad\qquad \left.\left. - \left(\frac{s^*(\varepsilon_t^{i,1} v^i(t), \Delta_{\mathrm{per}} v^i(t))}{\Delta_{\mathrm{per}} x^i(t)} \right)^2 \right), a^i_{\min} \right\}, \\ x^i(t_0^i) = 0,\ v^i(t_0^i) = v_0^i,\ t \geq t_0^i,\ i \in \mathcal{M} \end{cases}$$

where $s^*(s_1, s_2) = s_0 + s_1 T + \frac{s_1 s_2}{2\sqrt{a^i_{\max} b}}$ and

$$\Delta_{\mathrm{per}} v^i(t) = \varepsilon_t^{i,1} v^i(t) - \varepsilon_t^{i,2} v^{i-1}(t),$$
$$\Delta_{\mathrm{per}} x^i(t) = \varepsilon_t^{i,3} \Delta x^i(t) = \varepsilon_t^{i,3} (x^{i-1}(t) - x^i(t) - l_{i-1})$$

where l_i is the length of vehicle $i \in \mathcal{M}$. Moreover, $a^i_{\max} > 0$ is the maximal acceleration, and $a^i_{\min} < 0$ the minimal acceleration (i.e., maximal deceleration) of the i-th vehicle. The other parameters originate from the classic IDM model, and we refer to [3] for a detailed explanation. For the first vehicle $i = 1$, we set the interaction term $s^*(\varepsilon_t^{1,1} v^t(t), \Delta_{\mathrm{per}} v^1(t)) \cdot (\Delta_{\mathrm{per}} x^1(t))^{-1} := 0$ as there is no preceding vehicle.

B. Accidents

The stochastic processes $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2})$ and $(\varepsilon_t^{i,3})$, $i \in \mathcal{M}$, may be interpreted as different sources of errors. The classic IDM determines the velocity on the basis of the distance to the preceding vehicle and the approaching rate. In contrast, the IDMrm assumes that all these quantities are subject to perceptional errors. The perceived quantities are inputs to the calculation of the acceleration of each vehicle. Vehicle i uses for this computation instead of the true velocities $(v^i(t), v^{i-1}(t))$ of itself and the preceding vehicle the distorted values $(\varepsilon_t^{i,1} v^i(t), \varepsilon_t^{i,2} v^{i-1}(t))$; in addition, instead of the true distance to the preceding car $\Delta x^i(t)$ the randomly distorted value $\varepsilon_t^{i,3} \Delta x^i(t)$ is the third input to the calculation. There are no errors, as long as $\varepsilon_t^{i,1} = \varepsilon_t^{i,2} = \varepsilon_t^{i,3} = 1$. Our model is sufficiently flexible to admit many stochastic error processes. In our numerical case studies, we will assume that $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2})$ and $(\varepsilon_t^{i,3})$, $i \in \mathcal{M}$, are independent Ornstein-Uhlenbeck processes that randomly fluctuate around 1. This can be interpreted as noisy perception of the true values. Misperception can cause accidents.

An accident occurs when vehicles collide. Up to this point, their movement is described by the RODEs above. However, we assume that this is not the case anymore after a collision. If an accident occurs, collided vehicles will remain at their position for some time. Then they will be removed from the system. In the following, we will make this precise.

For $i \in \mathcal{M}$, let $A^i(t)$ denote the area of the road which is occupied by vehicle i at time $t > 0$. In the one-dimensional case this corresponds to the interval $A^i(t) = [x^i(t) - l_i/2, x^i(t) + l_i/2]$ where l_i denotes the vehicle's length and $x^i(t)$ is the position of the vehicle's midpoint. Formally, an accident occurs, if

$$\exists\ i, j \in \mathcal{M},\ i \neq j,\ \exists\ t > 0\colon A^i(t) \cap A^j(t) \neq \emptyset.$$

Now, if two vehicles collide, their velocities are immediately set to 0. Depending on the traffic constellation, further vehicles may crash into an existing collision or perform a safe emergency braking maneuver. We assume that at the time of the first collision, an exponentially distributed waiting time $t_{\mathrm{removal}} \sim \mathrm{Exp}(\gamma)$, $\gamma > 0$, is triggered; as this time has passed, all vehicles that collided disappear from the model. The expected waiting time until vehicles are removed is $\mathbb{E}\,(t_{\mathrm{removal}}) = \gamma^{-1} > 0$. We note that other accidents may occur at different locations in the system; the removal time at different locations is triggered independently in each case.

In summary, if an accident occurs, the one-lane road is blocked and a traffic jam emerges. Later – after a random time t_{removal} – vehicles that collided are removed from the road; remaining vehicles will continue their journey, and the traffic jam dissipates.

IV. Case Studies

In this section, our approach will be illustrated in the context of two traffic scenarios: In the first scenario, a one-dimensional road segment is considered. The vehicles enter at the beginning of the road segment, the origin, and disappear at its other end. We analyze the evolution of traffic over a fixed period of time and focus on safety and efficiency. The second scenario describes a more complex situation: a t-junction composed of two intersecting one-lane road segments.

Our traffic model is capable of capturing heterogeneous vehicles. Note that each vehicle is endowed with its own set of parameters and associated stochastic processes. This allows to model individual driving behavior and corresponding error patterns. In this paper, we focus on a simplified version of the model with homogeneous traffic participants, highlighting the effects of varying parameters. Misperception is captured by the processes $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2}), \ldots, (\varepsilon_t^{i,5})$, $i \in \mathcal{M}$, which we assume to be Ornstein-Uhlenbeck processes fluctuating around 1.

We fix a terminal time $T_{\text{sim}} > 0$ for the traffic simulation. Vehicles are consecutively enumerated by $1, 2, 3, \ldots$ and enter each lane-segment at its origin paying attention to existing traffic. The exact procedure will be described below, but we already stress at this point that due to the randomness of traffic flow also the collection of vehicles $\mathcal{M}$ that are generated until terminal time T_{sim} is random. We simulate the traffic system and compute statistics characterizing safety and efficiency from $m \in \mathbb{N}$ independent simulation runs.

Measure of Efficiency. As a measure of efficiency for the traffic system we choose traffic *flow* per time unit, measured at position d (in our simulations, we choose d as the end of the road):

$$Q = \frac{\text{card}\{j \in \mathcal{M} : \exists\, t \leq T_{\text{sim}} : x^j(t) = d\}}{T_{\text{sim}}}.$$

Here, *card* denotes cardinality. In the following, we denote sample averages that we compute from our simulation runs by a circumflex. For example, the sample average of the flow is $\hat{Q}$.

Measure of Safety. A measure of traffic safety is the number of accidents per time unit. The term *accident* refers to an event where at least two vehicles collide. If more vehicles crash into an existing collision, this does not create a new accident according to our convention.

Recall that the area occupied by vehicle $i \in \mathcal{M}$ at time $t \geq 0$ is denoted by $A^i(t)$; additionally, for $M \subseteq \mathcal{M}$ we define $A^M(t) := \bigcup_{i \in M} A^i(t)$. The number of accidents per time unit, denoted by f_{acc}, is given by

$$\begin{aligned} f_{\text{acc}} = \frac{1}{T_{\text{sim}}} \cdot \text{card}\{\emptyset \neq M \subseteq \mathcal{M} :& \\ \exists\, t \leq T_{\text{sim}}\, \forall\, i \in M : A^i(t) \cap A^{M \setminus \{i\}}(t) \neq \emptyset& \\ \text{and } \forall\, t \leq T_{\text{sim}} : A^M(t) \cap A^{M^c}(t) = \emptyset\}& \end{aligned}$$

where M^c denotes the complement of M. The first condition ensures that all vehicles in M collide, the second that all vehicles involved in the accident are identified. The corresponding sample average is denoted by $\hat{f}_{\text{acc}}$.

A. One-Lane Road Segment

This scenario consists of a segment of a one-lane road, a one-dimensional line $[0, L]$ of length $L = 2{,}000\,\text{m}$. Vehicles are generated at the origin and are removed when they reach the end of the road. Their generation is defined by the following algorithm: Vehicles are created deterministically with a constant demand (here, $1{,}500\,{}^{\text{veh}}/_{\text{h}}$), if there is enough space available at the beginning of the road. More precisely, a vehicle $i \in \mathcal{M}$ may be generated according to the desired demand, if there is no other vehicle in the first $7.5\,\text{m}$ of the road (which equals the vehicle length plus an additional safety margin of $1.5\,\text{m}$); otherwise, the generation of the new vehicle is delayed until this condition is satisfied. The initial velocity of any new vehicle matches the velocity of the preceding vehicle. In summary, the initial generation of vehicles avoids artificial accidents; instead, accidents may be caused by random misperception at a later point in time somewhere on the lane.

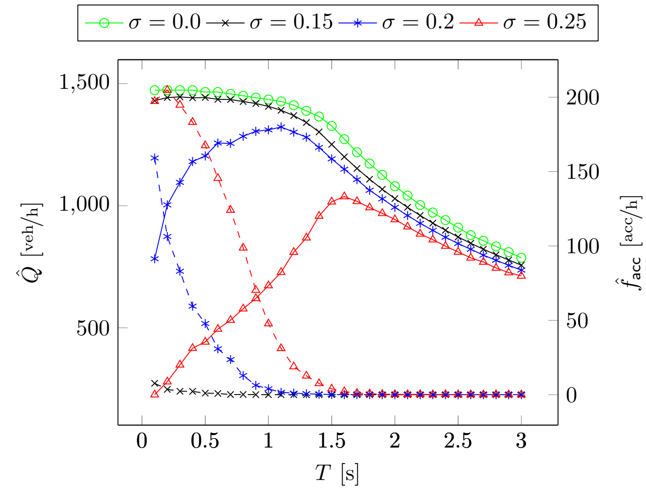

Fig. 2. Averaged flow and number of accidents for varying T and fixed σ with $1{,}000$ independent simulations for each parameter combination: Dashed lines correspond to number of accidents, solid lines to flows.

The remaining parameters used for our simulation are given in Table I. Traffic is simulated for a duration of $T_{\text{sim}} = 600\,\text{s}$ according to IDMrm. The error processes $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2}), (\varepsilon_t^{i,3})$ are independent and identically distributed copies of an Ornstein-Uhlenbeck process with $\alpha = \beta = 1$ and different values of σ; in order to guarantee that we may observe sufficiently many accidents in our small-scale example, we choose relatively high volatilities. Smaller volatilities require longer simulations and larger roads to observe a sufficient number of accidents which increases the computational effort. This does not alter the methodology, and the corresponding case studies may be analyzed in the future.

In the analysis of the model we focus on the effect of a varying error volatility σ and a varying time headway T. The volatility σ is a measure for the size of the random

TABLE I
PARAMETER CHOICE FOR THE SCENARIOS

Scenario	$a_{\max}$	v_{d}	δ	$a_{\min}$	s_0	T	b	l	α	β	σ	γ
A	2.0	15	4	-3.5	1.2	$\cdot$	1.67	6	1	1	$\cdot$	1/60
B	2.0	10	4	-3.5	2.0	1.5	1.67	6	1	1	$\cdot$	1/300

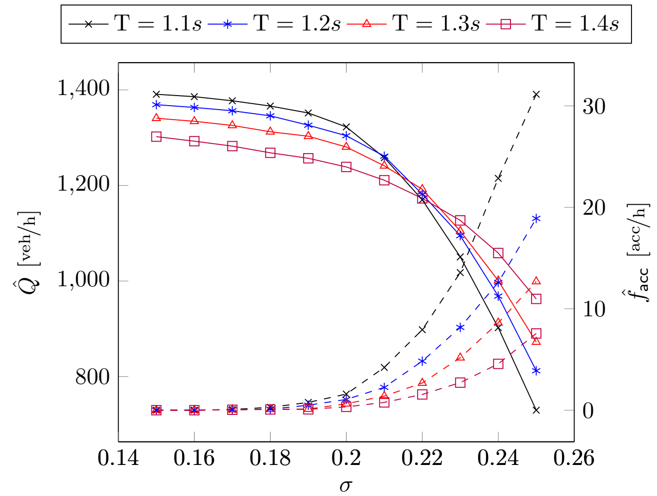

Fig. 3. Averaged flow and number of accidents for varying σ and fixed T with $1,000$ independent simulations for each parameter combination: Dashed lines correspond to number of accidents, solid lines to flows.

misperception in the model. The time headway T is a parameter in the IDMrm that influences the safety distances. In the absence of random misperception, the bigger the time headway, the greater is the distance between vehicles and the lower the traffic flow. If errors are present, a larger time headway will decrease the number of accidents. Since a large number of traffic accidents may also decrease flow, we expect that the dependence of flow on time headway is not always monotone anymore.

We analyze the behavior of the system after the first vehicle has reached the end of the road. Both flow and the number of accidents are random. We display their sample averages (approximating their expectations) in Fig. 2 for varying T and different fixed values of σ. The case $\sigma = 0$ corresponds to no misperception with no accidents. As a consequence, minimizing T leads to the maximal flow – almost equal to the demand of $1,500\,{}^{\text{veh}}/_{\text{h}}$. With increasing σ, we observe decreasing flow. The rational is that accidents lead to traffic congestion which decreases flow. If we fix σ but vary the time headway T, we can find a T that maximizes the flow. This shows the interplay between safety and efficiency: Accidents decrease the traffic flow. A larger time headway T decreases the number of accidents, but, if there are only few accidents, also decreases flow. Thus, for small T, flow increases with increasing T due to a decreasing number of accidents, but for large T flow decreases with increasing T.

This tradeoff can also be observed in Fig. 3. The figure displays the sample averages of both flow and the number of accidents for varying σ and different fixed values of T. Of course, with increasing σ, $\hat{f}_{\text{acc}}$ increases and $\hat{Q}$ decreases. The key point is to observe that the flow curves intersect! At a certain level of misperception (with too many accidents occurring), it becomes more efficient to select a larger time headway T that decreases the number of accidents and increases efficiency.

B. Left-Turning on T-Junction

In Scenario B, we analyze traffic at a t-junction with vehicles that turn left. We focus on the simplified setting shown in Fig. 4: Vehicles on the bottom lane always turn left, following the green path – while vehicles on the top lane never turn, following the black path. On an abstract level, this scenario can be decomposed into two one-lane road segments with the additional property that these segments intersect. By creating an intersection, we introduce conflicts: vehicles can collide at the t-junction. We assume that left-turning vehicles give way to oncoming vehicles on the upper lane. For example, they might need to stop at a certain point (denoted by x_{stop}; cf. Fig. 4) in order to let the conflicting vehicles pass the junction (i.e., reach x_{exit}; cf. Fig. 4). We assume that vehicles on the upper lane insist on their right of way and are oblivious to oncoming traffic, i.e., they do not react.

In this case study, the horizontal lane has a length of $300\,\text{m}$ with the junction placed at $97.5\,\text{m}$. Green and black path intersect at $94.5\,\text{m}$ and x_{stop} is located at $85.0\,\text{m}$. We generate vehicles similarly to Scenario A, but instead of assuming a fixed rate we create them with an exponentially distributed headway with mean $500\,{}^{\text{veh}}/_{\text{h}}$ on the upper lane and mean $200\,{}^{\text{veh}}/_{\text{h}}$ on the lower lane. This stochastic generation of vehicles yields interesting dynamics with vehicles on the lower lane reacting to the upper lane: vehicles need to wait for emerging gaps in order to turn; gaps occur at random times.

We implement IDM on the upper lane with the same adjustments as for IDMrm, i.e., we implement IDMrm with perfect perception. We do the same on the lower lane, but introduce as an additional feature conflict detection and reaction. Random misperception could be introduced at different points of the model, but we focus in this paper only on errors in the conflict detection and reaction which may create accidents in the context of left-turning maneuvers.

We heuristically describe the implementation of the conflict detection and reaction for a turning vehicle i and one oncoming vehicle j. Vehicle i follows the green path according to the implemented car-following model. Additionally, as a turning vehicle it can detect conflicts. Turning vehicles are aware of oncoming vehicles and extrapolate the trajectories of potentially conflicting vehicles based on observations of their movement. They also extrapolate their own trajectories. Vehicle i checks if braking is necessary in order to turn safely. For this purpose, a distance $\hat{d}^{ij}(t)$ of vehicle i to the approaching vehicle j is estimated. The situation is classified as a conflict, if the estimate $\hat{d}^{ij}(t)$ is smaller than a given safety threshold d_{s} at some point in time. We refer to [12] for a detailed description of conflict detection. If no conflict arises, the vehicle turns. Otherwise, it decelerates according to the following algorithm. To simplify the notation, we omit dependency on t. First, we compute a naive deceleration a_{naive} such that vehicle i stops at x_{stop} with a constant (negative) acceleration given by $a_{\text{naive}} = -(v^i)^2 / \left(2\left(x_{\text{stop}} - x^i\right)\right)$. For this, the vehicle needs a time of $t_{\text{naive}} = -v^i/a_{\text{naive}}$. We let $\hat{t}^j$ be the time vehicle j needs to reach x_{exit} which is computed by numerically inverting the extrapolated trajectory. If $\hat{t}^j > t_{\text{naive}}$, vehicle i sets a_{naive} as its acceleration, stops at x_{stop} and waits until it can turn safely. If $\hat{t}^j \leq t_{\text{naive}}$ (i.e., vehicle i does not need to stop since vehicle j will have passed the junction by that time), we choose the following acceleration

$$a^{i,j}_{\text{smooth}} = \left(\frac{x_{\text{stop}} - x^i}{\hat{t}^j} - v^i \right) \frac{2}{\hat{t}^j}$$

which is determined such that vehicle i reaches x_{stop} when vehicle j arrives at x_{exit}. Stopping is not necessary in this situation. These computations are carried out for all possibly conflicting vehicles, and the minimal resulting acceleration is chosen, while also taking into account possible car-following behavior.

Random misperception is implemented as follows: For the conflict detection, vehicle i extrapolates trajectories and computes an estimated distance $\hat{d}^{ij}(t)$ from the conflicting vehicle. Only this distance measure is the quantity that triggers i's reaction as described above. The extrapolation of i's own trajectory is based on its velocity $v^i(t)$. We assume that both $v^i(t)$ and $\hat{d}^{ij}(t)$ are subject to misperception: We implement two independent and identically distributed Ornstein-Uhlenbeck processes $(\varepsilon^{i,4}_t)_{t\geq 0}$ and $(\varepsilon^{i,5}_t)_{t\geq 0}$ (as in Scenario A) to distort these values, i.e., replacing them with $\varepsilon^{i,4}_t v^i(t)$ and $\varepsilon^{i,5}_t \hat{d}^{ij}(t)$ where $\hat{d}^{ij}(t)$ is already computed on the basis of the misperceived velocity $\varepsilon^{i,4}_t v^i(t)$. As a consequence, both the conflict detection and the conflict reaction may be erroneous.

We begin with our measurement when the first vehicles reach the ends of both lanes. As before, we simulate traffic for $600\,\text{s}$ and implement an exponentially distributed removal time t_{removal} for vehicles that collided. We evaluate the dependency of flow and number of accidents on safety distance d_{s} and volatility σ of the Ornstein-Uhlenbeck processes.

In Fig. 5 we fix $d_{\text{s}} = 1.5\,\text{m}$ and present the effect of increasing volatility on number of accidents and traffic flow for both lanes. The figure displays similar phenomena as Fig. 3, confirming our expectation that random misperception causes accidents. Compared to Scenario A, we observe fewer accidents which is due to more conservative parameter choices in the implementation of the turning maneuver (cf. Table I). However, one can still see that with higher volatility more accidents occur, causing a decline of traffic flow.

Next, we investigate the impact of d_{s} for fixed volatilities σ. We analyze the traffic flow on the lower lane (see Fig. 6) and on the upper lane (see Fig. 7) separately. The case $\sigma = 0$ corresponds to no misperception; accidents may still occur in this case due to extrapolation errors for d_{s} too small. For different values of σ, traffic on the lower lane exhibits a similar behavior as in Scenario A, see Fig. 2: If d_{s} is too small, many accidents occur such that traffic flow is impaired. With increasing d_{s}, the number of accidents decreases. This initially improves the traffic flow, but if d_{s} becomes too large, traffic flow again decreases. In this situation, vehicles cannot easily find gaps in the oncoming traffic that permit turns. On

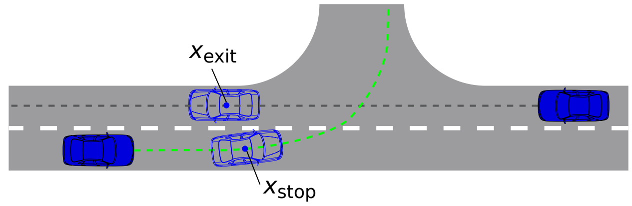

Fig. 4. Simplified left-turning scenario on t-junction.

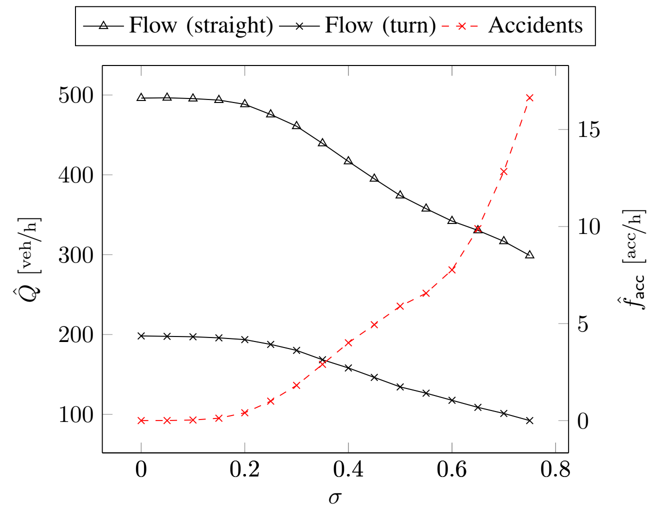

Fig. 5. Averaged traffic flow (turn and straight) and number of accidents for varying σ and $d_{\text{s}} = 1.5\,\text{m}$ with $10,000$ independent simulations.

the upper lane, in contrast, flow is strictly increasing with increasing d_s. This is not surprising, since the traffic flow on the upper lane is only distorted, if turning vehicles cause accidents.

V. Conclusion & Future Research

We introduced a traffic model that admits accidents. The accidents are caused by random misperception, a type of error that affects both human drivers and autonomous vehicles. The simulation model admits a characterization of the tradeoff between safety and efficiency of traffic systems. While empirical data on the traffic systems of the future are not available yet, our causal stochastic model produces simulated data that provide guidance to the design and risk management of future traffic systems.

In our case studies, we studied homogeneous traffic participants and one particular error pattern, modeled by independent Ornstein-Uhlenbeck processes. However, our approach can also capture heterogeneity, i.e., multiple driving styles and error types. In particular, the model can be used to analyze effects of systems that include both human drivers and autonomous vehicles. Such traffic systems will be relevant in the near future. Moreover, one could try to generalize the model to include the effects of V2V-communication that might be subject to random errors.

We have demonstrated that optimality in terms of traffic flow does typically not imply that accidents are absent; accidents cause harm to the society. Future research should define

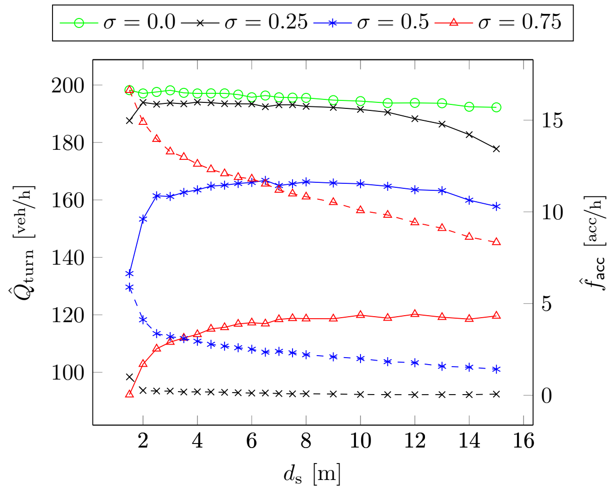

Fig. 6. Averaged turn flow (lower lane) and number of accidents for varying d_s and fixed σ with $10,000$ independent simulations for each parameter combination: Dashed lines correspond to number of accidents, solid lines to flows.

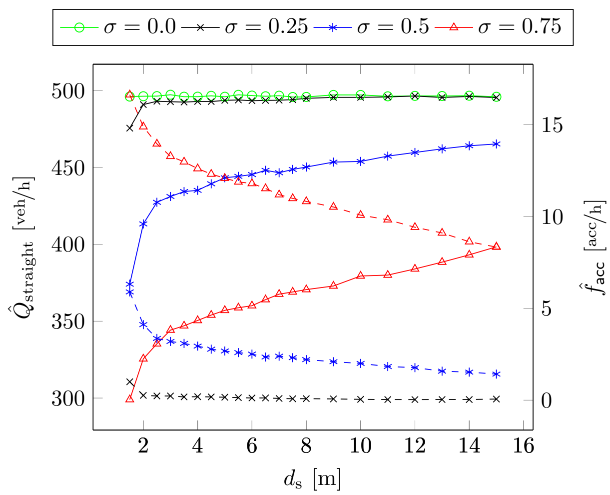

Fig. 7. Averaged straight flow (upper lane) and number of accidents for varying d_s and fixed σ with $10,000$ independent simulations for each parameter combination: Dashed lines correspond to number of accidents, solid lines to flows.

and include the cost of accidents to the analysis. This requires a model of the severity of accidents, an issue that was neglected in the current paper. From a computational point of view, the efficiency of the simulation might be improved by applying well-designed variance reduction techniques. Since accidents are rare events, variance reduction techniques for rare-event simulation, such as importance sampling, might be promising.

Acknowledgment

The implementation of the simulation models is partially based on results of the MODIS research project that was financially supported by the German Research Foundation (DFG).

The authors would like to thank Kerstin Weske (Institute of Probability and Statistics) for her involvement in the project initiation.

We are grateful for helpful remarks of the editors and referees. We thank Rolf Harren, Frederik Hesse, Per-Johan Horgby, Johannes Langner and Sebastian Reddemann for helpful comments and discussions.

Marcel Kleiber gratefully acknowledges financial support by Hannover Insurance School.

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