The Impact of Insurance Premium Taxation

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Abstract

In many countries insurance premiums are subject to an insurance premium tax that replaces the common value-added tax (VAT) used for most products and services. Insurance companies cannot deduct VAT payed on inputs from premium tax; also corporate buyers of insurance cannot deduct premium tax payments from VAT on their outputs. Such deductions would be allowed, if insurance premiums were subject to VAT instead of insurance tax.

In the current paper, we investigate the impact of the premium tax on insurance companies, insurance holders and government revenues from multiple perspectives. We explicitly compare tax systems with premium tax and tax systems that allow deductions. We find that the competitiveness of corporate buyers of insurance, the ruin probabilities of insurance firms and their solvency capital are hardly affected by the tax system. In contrast, the tax system has a significant influence on the cost of insurance, insurance demand, government revenues and the profitability of insurance firms.

1 Introduction

In many countries insurance premiums are subject to insurance premium tax that replaces the common value-added tax (VAT) used for most products and services. This is, for example, mandatory according to EU-law. In contrast to VAT, premium tax does not permit any deductions: first, insurance companies cannot deduct VAT payed on inputs from premium tax; second, corporate buyers of insurance cannot deduct their premium tax payments from VAT on their outputs, although the insurance contracts are an input to their production. As a consequence, insurance premium tax leads to a higher taxation than VAT if the same tax rate is applied. In the current paper, we investigate the impact of premium tax on insurance companies, insurance holders and government revenues from multiple perspectives.

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Subject to premium tax are insurance premiums only. In the case of insurance companies, these are approximately equal to the total revenues of these firms. VAT – in contrast – is not charged on revenues, but on the value added which is smaller than revenues.

Premium tax rates and VAT rates vary across countries. Tax rates may also be different for different types of products. Information on tax rates for individual countries and products can be found in European Commission (2017), Insurance Europe (2016), and Bundesministerium der Justiz und für Verbraucherschutz (2017). In Germany the VAT rate and the premium tax rate coincide for most types of products and are generally both equal to 19%.

In absolute terms, government revenues from VAT are much larger than revenues from premium tax due to a larger tax base. In 2015 VAT revenues in Germany were 159,015 million EUR – corresponding to 23.6% of total tax revenues; premium tax revenues during the same year were equal to 12,419 million EUR, i.e. 1.8% of total tax revenues or 7.8% of VAT revenues, see Bundesfinanzministerium (2016).

On the individual level of both the providers and buyers of insurance, premium tax may lead to higher total tax payments. We quantify this effect in Section 2. As already explained, on the one hand a provider of insurance cannot deduct VAT paid on input goods from premium tax. On the other hand commercial buyers of insurance cannot deduct the incurred premium tax from VAT on their outputs. While Section 2 provides an analysis from the point of view of individual tax payers, Section 3 calculates the impact on overall tax revenues. More specifically, a tax system with insurance tax is compared to one in which insurance tax is replaced by VAT. If the VAT rate was unchanged, total tax revenues would decrease by 16 billion EUR. An equivalent VAT rate of 89.2% is computed that leads to the original total tax revenues.

After analyzing the basic differences between VAT and premium tax, we provide in Sections 4 – 7 a broader perspective on the topic by comparing the impact of different tax systems on insurance demand, the competitiveness of corporate buyers of insurance, ruin probabilities of insurance firms, and solvency capital. In Section 4 we model corporate buyers of insurance as utility maximizers that can choose their optimal level of insurance. The total cost of insurance depends on the tax system that is implemented. We provide case studies that illustrate potential consequences on the demand for insurance. These show that a change in the tax system from insurance to value-added tax increases the insurance demand. Section 5 investigates the competitiveness of corporate buyers of insurance. In contrast to Section 4 it is assumed that the amount of insurance that is bought is constant, but that its cost depends on the tax system. If an insurance tax is replaced by a VAT with the same rate, the costs are effectively reduced. If these savings are completely passed on to the buyer of the output products, the demand for these products increases. This is explicitly quantified in two numerical case studies. While Section 5 assumes that tax savings are used to reduce the price of output products, Sections 6 and 7 suppose that savings are retained by the insurance company. In Section 6 we generalize the classical Cramér-Lundberg model by including tax payments in order to study the impact on ruin probabilities. Finally, Section 7 computes how solvency capital requirements change that are e.g. implemented under the regulatory framework of Solvency II or the Swiss Solvency Test. In summary, we find that the competitiveness of corporate buyers of insurance, the ruin probabilities of insurance firms and their solvency capital are hardly affected by the tax system. In contrast, the tax system has a significant influence on the cost of insurance, insurance demand, government revenues and the profitability of insurance firms. Section 8 concludes with a discussion and suggestions for further research.

Literature. Holzheu (1997) and Holzheu (2000) suggest an accounting methodology in order to compute basic quantities that characterize the impact of a premium tax. Sections 2 & 3 build on this methodology. Our sections are, however, based on current data and constitute a necessary prerequisite for the other parts of our paper. Straubhaar (2006) provides a qualitative analysis of the impact of an increased premium tax that is complemented by a regression analysis in order to obtain quantitative estimates. Schrinner (1997) qualitatively discusses the impact of premium tax and provides related accounting figures besides a preliminary economic analysis.

2 Impact on Insurance Companies and Insurance Holders

Insurance premium tax and value-added tax lead to different tax expenses related to insurance services. To begin with, we provide a descriptive analysis on the basis of aggregate accounting data that quantifies the impact of different tax systems. Our computations follow the methodology described in Holzheu (1997) & Holzheu (2000), using data from 2011 – 2015 provided in BaFin (2011-2015).

Impact on Insurance Companies. We consider an insurance company with earnings of *gross premiums* denoted by $\pi \in \mathbb{R}$. Gross premiums are before the deduction of reinsurance. For the purpose of our comparisons, we define *taxed premiums* as the sum of gross premiums π and the tax that is charged from the policyholders for their insurance contracts, i.e. either premium tax or value-added tax – depending on the (real or hypothetical) tax system that we consider.

We denote the prevailing *VAT rate* by $\tau_{VAT} \geq 0$. The input of the production of insurance contracts is always taxed according to VAT. *Taxed input*[1] $\bar{L}$ and *untaxed input* L can thus be converted into each other:

$$\bar{L} = (1 + \tau_{VAT})L.$$

We define the *rate of input* α as the ratio of untaxed input and untaxed gross premiums earned:

$$\alpha = \frac{L}{\pi}.$$

Tax legislation in many countries typically prohibits the deduction of VAT paid on inputs from premium tax, but would allow a deduction if instead VAT was also paid on outputs. The amount that would be deducted in this case equals

$$\bar{L} - L = \tau_{VAT}\alpha\pi. \tag{1}$$

We now estimate this quantity that is typically not directly reported by insurance companies. Untaxed inputs can roughly be estimated as gross premiums earned plus capital income minus total losses and costs including taxes. For Germany, the required data were obtained from the annual reports of *Bundesanstalt für Finanzdienstleistungsaufsicht (BaFin)*.

Example 2.1. *On the basis of BaFin-data for the years 2011 – 2015 (see BaFin (2011-2015)), the mean rate of input*[2] *equals* $\alpha = 5.2\%$. *With* τ_{VAT} *=19%, this implies a difference of untaxed and taxed inputs equal to* $\bar{L} - L \approx 0.99\% \cdot \pi$.

Equivalent Value-Added Tax Rate. The current tax system charges VAT on the production inputs of insurance firms, but premium tax on their outputs, i.e. on insurance contracts. A deduction of VAT on inputs from premium tax is not permitted. What is the hypothetical VAT rate on the value added generated by the insurance contracts that leads to the same tax payments as insurance tax? We call this counterfactual VAT the equivalent value-added tax. We stress that the VAT rate on all other goods and services remains unchanged in this gedankenexperiment. We simply analyze a modified basis of assessment of the tax that is charged on insurance contracts, holding the tax revenue constant. In the case of premium tax, the basis of assessment are the premiums; in the case of the equivalent VAT, the basis of assessment is the value added generated by the insurance industry.

[1]In this paper, taxed variables are marked with a bar $\bar{\ }$.

[2]Input ratios were estimated on the basis of an aggregated stylized income statement of insurance companies provided by BaFin. Input ratios were computed for single years, then added and finally averaged over time. For the detailed computation we refer to Appendix B.

We denote the *premium tax rate* by $\tau_{PT} \geq 0$ and the insurance's value added before taxes by $\widetilde{W}$. The value added before taxes is estimated as the sum of acquisition costs and administrative expenses, profits before taxes and changes in equalization provisions and similar provisions. In order to account for the fraction of the value added that is indirectly generated by reinsurers' share of gross premiums earned we add the difference between the gross technical result and net technical result. From BaFin-data 2011 – 2015[3] we compute

$$\widetilde{W} \approx 31.3\% \cdot \pi.$$

The equivalent value-added tax rate that leads to the same tax revenue can be calculated by the change of basis of assessment equation,

$$\widetilde{\tau_{VAT}} \widetilde{W} = \tau_{PT} \pi \;\Leftrightarrow\; \widetilde{\tau_{VAT}} = \tau_{PT} \cdot \frac{\pi}{\widetilde{W}}.$$

Example 2.2. *For many types of insurance contracts the premium tax rate in Germany equals* $\tau_{PT} = 19\%$. *With* $\pi/\widetilde{W} \approx 1/31.3\% = 3.19$ *as calculated above, we obtain an equivalent VAT rate of*

$$\widetilde{\tau_{VAT}} \approx 60.7\%.$$

Remark 2.3. *Value added before taxes varies among different lines of insurance. Equivalent VAT rates are displayed in Table 1.*[4]

Class	**Value added ratio**	**Equivalent VAT rate**
Accident	39%	49%
Public liability	37%	51%
Car total	21%	89%
Defense	37%	51%
Fire	35%	55%
Household	43%	45%
Residential building	32%	59%
Credit and guarantee	37%	51%
Total	31%	61%

Table 1: Equivalent VAT rates for different lines of insurance.

Impact on Insurance Holders. Next, we consider a hypothetical tax system in which VAT can be deducted from premium tax, and premium tax from VAT. In this case, the basis of assessment of the premium tax are the gross premiums earned, but counterfactual deductions are admissible. We assume that all tax savings are passed to a corporate buyer of insurance contracts. The latter are treated as an input good to the buyer's production, allowing for a deduction of incurred premium tax from VAT on outputs of the corporate customer. Total tax savings can thus be decomposed into two components: a) the VAT on the input goods of insurance firms deducted from premium tax, b) premium tax deducted from the VAT on the output goods of the corporate buyer of insurance.

We compute the size of these tax savings. For this purpose, we denote the total revenue (or business volume) of the corporate insurance holder by $U \in \mathbb{R}$ and her untaxed cost of insurance by $V \in \mathbb{R}$. The insurance ratio of the company is defined as

$$\beta := \frac{V}{U} \in [0, 1]. \tag{2}$$

[3]Value added was calculated for every year, summed up and averaged over time. For the detailed computation we refer to Appendix B.

[4]For the detailed computation of the quantities we refer to Appendix B.

Lemma 2.4. *If VAT and premium tax can be deducted from each other, if taxes on outputs are higher than on inputs and if all tax savings are passed on to a corporate buyer of insurance, then this firm has tax savings of*

$$E = (\tau_{VAT}\ \alpha + \tau_{PT})\, \beta U. \tag{3}$$

Proof. The savings are given by eq. (1) plus the input tax reduction of the company for insurance products., i.e. $E = \tau_{VAT}\ \alpha V + \tau_{PT}\ V$. □

Example 2.5. *Setting the input ratio to α =5.2% as in Example 2.1 and the insurance ratio to β =0.5%,*[5] *we obtain*

$$E = 0.1\% \cdot U,$$

i.e. the total savings are only approximately 10 basis points of the revenues of the company. The parameter β depends on the industry sector of the corporate insurance holder. According to Swiss Re, sigma No 5/2012 (sigma (2012), p. 17) it varies between 0.1% *and* 1.4% *for different US industries.*

In summary, we estimated for insurance firms in Germany that their mean rate of input is about 5% of gross premiums, implying a difference between untaxed and taxed inputs of about 1% of gross premiums. For a premium tax of 19%, an equivalent VAT rate depends on the line of insurance and ranges from about 50% for accident insurance to about 90% for car insurance. Finally, we considered a counterfactual tax system in which VAT and premium tax can be deducted from each other and estimated for Germany that tax savings of corporate policyholders would amount to about 10 basis points of their total revenues.

3 Impact on Tax Revenues

We will now discuss the impact on total tax revenues, if premium tax is replaced by VAT. First, we compute the modified tax revenues. Second, we calculate an equivalent VAT which leads to the same total tax revenues. Our findings build on the results of the previous section. The methodology is motivated by Holzheu (2000), p. 76ff.

Comparison of Tax Revenues. Let Π be the total national untaxed gross insurance premiums earned, $\widetilde{W}$ the total value added before taxes of the corresponding insurance companies, and α their input ratio. The tax revenue S related to insurance contracts in a tax system with premium tax can be split into three parts:

(i) VAT of insurance companies on their inputs: This amount cannot be deducted from premium tax. It can be computed according to eq. (1).

(ii) Premium tax.

(iii) VAT on taxed insurance premiums: The costs of the outputs of corporate buyers of insurance are increased by the premium tax. This is implicitly reflected in their prices and leads to additional value-added tax revenues.

Total tax revenues are given by adding up the three parts:

$$S = (\tau_{VAT}\alpha + \tau_{PT}) \cdot \Pi + \tau_{VAT} \cdot (1 + \tau_{PT}) \cdot \Pi_G.$$

Here, Π_G denotes the untaxed insurance premiums of corporate insurance holders. Policies of private customers are included in Π but not in Π_G, since they are not indirectly charged with additional VAT.

[5]As a rough approximation of β we use an estimate that is provided in Swiss Re, sigma No 5/2012 (sigma (2012), p. 16) for the US market. Since the main purpose of Example 2.5 is to provide an estimate of the order of the impact of a modified tax system on corporate insurance costs, precise knowledge of β is not required.

Suppose now that insurance premiums are not subject to a premium tax but to a value-added tax. In this case, these taxes are fully deductable and double-taxation is avoided. The total relevant tax revenue thus amounts to $\tilde{S} := \tau_{VAT}\widetilde{W}$. In particular, if we assume that $\tau_{PT} \geq \tau_{VAT}$ and $\widetilde{W} < \Pi$, then

$$\tilde{S} = \tau_{VAT}\widetilde{W} < \tau_{PT}\Pi < S.$$

Changing the tax system from premium tax to VAT thus leads to lower total tax revenues, if the corresponding tax rates are equal.

Equivalent Value-Added Tax Rate. As before, we compute an equivalent VAT rate $\widetilde{\tau_{VAT}}$ that leads to the same tax revenues, but now also incorporates taxes on premium tax paid by corporate policyholders. As in the previous section we assume that the VAT rate on all other goods and services remains unchanged in this thought experiment. We only focus on those tax revenues that are directly related to insurance contracts as explained above.

Lemma 3.1. *We denote by $g := \frac{\Pi_G}{\Pi}$ the ratio of untaxed corporate insurance premiums over total untaxed insurance premiums. The equivalent value-added tax rate $\widetilde{\tau_{VAT}}$ is given by*

$$\widetilde{\tau_{VAT}} = (\tau_{VAT}\alpha + \tau_{PT} + \tau_{VAT}(1+\tau_{PT})g) \cdot \frac{\Pi}{\widetilde{W}}.$$

Proof. The result follows immediately from the condition $\widetilde{\tau_{VAT}}\widetilde{W} = S$. □

Example 3.2. *The average value added before taxes of German insurance firms during the period 2011 – 2015*[6] *amounts to*

$$\widetilde{W} \approx 31.3\% \cdot \Pi.$$

If we assume that the fraction of premiums of corporates is g =35%,[7] *we obtain for $\tau_{VAT} = \tau_{PT} = 19\%$ and $\alpha = 5.2\%$*[8] *an equivalent value-added tax rate of*

$$\widetilde{\tau_{VAT}} \approx 89.2\%.$$

Finally, let us consider a modification of the tax system in which premium tax is replaced by VAT. This would, in particular, imply that both VAT on insurance companies' inputs and VAT on premium tax paid by corporate customers are deductible. For the purpose of illustrating the size of this effect, we suppose that the German premium tax of 19% *is replaced by VAT of* 19%. *This would decrease total German tax revenues by approximately*

$$S - \tau_{VAT} \cdot \widetilde{W} = (\tau_{VAT}\alpha + \tau_{PT} + \tau_{VAT}(1+\tau_{PT})g - \tau_{VAT} \cdot 31.3\%) \cdot \Pi = (27.9\% - 5.9\%) \cdot \Pi = 22\% \cdot \Pi.$$

Taking $\Pi \approx$ EUR 75 billion (corresponding to German gross premiums earned in 2015), tax revenues would decrease by approximately EUR 16 billion, if the tax system was changed.

4 Impact on Insurance Demand

The current German premium tax leads to an additional tax burden for insurance contracts. In this section we investigate the impact of the tax system on insurance demand. Insurance demand is endogenously modeled in a classical expected utility framework. For proportional insurance, we compute the optimal demand maximizing the expected utility of the policyholder.

[6] Compare Footnote 3.

[7] This number quantifies the fraction for Germany in 2010 according to sigma (2012), p. 10, Table 3. 35% of the total non-life premium income was generated by corporate buyers of insurance contracts.

[8] See Example 2.1.

Model 4.1 (Insurance Demand)**.** *We consider a proportional insurance contract over a fixed time horizon. The initial wealth of the insurance holder is denoted by $w > 0$. Over the time interval, the insurance holder incurs a* random loss $X \in L^1(\mathbb{R}_+)$ *where $L^1(\mathbb{R}_+)$ denotes the space of integrable real-valued random variables on some probability space $(\Omega, \mathcal{F}, P)$ with values in $\mathbb{R}_+$. The insurance contract is characterized by the parameter $\nu \in [0,1]$ which is the fraction of the loss that is covered by the insurance. The premium for* full insurance *is $\pi \in \mathbb{R}_+$; the premium for* partial insurance *of a fraction ν of the loss X is $\nu \cdot \pi$.*

The terminal endowment of the insurance holder as a function of ν is

$$X_\nu = w - X + \nu(X - \pi) = (1-\nu)(w - X) + \nu(w - \pi).$$

Buyers of insurance can choose the fraction ν according to their preferences. This fraction is computed as the solution to a utility maximization problem of the policyholder, see e.g. Chapter 2 in Föllmer & Schied (2011).

As a first example, we consider a Bernoulli utility function with *constant absolute risk aversion* (CARA). This function has the form $u_1^\kappa(x) = 1 - e^{-\kappa x}$ with $\kappa > 0$. Another example is a Bernoulli utility function with *hyperbolic absolute risk aversion* (HARA), given by $u_2^\lambda(x) = \frac{1}{\lambda}x^\lambda$ for $\lambda \in (0,1)$. The limiting case $\lambda = 0$ corresponds to logarithmic utility. The Arrow-Pratt-coefficients of absolute risk aversion are κ for u_1^κ and the hyperbolic function $x \mapsto (1-\lambda)/x$ for u_2^λ, explaining the terminology. In the case of HARA utility, we will always assume that $X \leq w$, for logarithmic utility $X < w$.

Problem 4.2 (Expected Utility Maximization)**.** *Let $S \subseteq \mathbb{R}$ be convex and assume that $u : S \to \mathbb{R}$ is a Bernoulli utility function, i.e. a function that is strictly concave, strictly increasing and continuous on S. Suppose that the support* supp X_ν *is contained in S and that $u(X_\nu)$ is integrable with respect to P for all $\nu \in [0,1]$. Then the optimal insurance contract is characterized by the maximizer $\nu^* \in [0,1]$ of the expected utility*

$$\nu \mapsto \mathbb{E}\left[u(X_\nu)\right].$$

A necessary condition for an interior solution $\nu \in (0,1)$ is given by the *first order condition*

$$\frac{\partial}{\partial \nu}\mathbb{E}\left[u(X_\nu)\right] = 0.$$

We compare different tax regimes for two examples of loss distributions, a Bernoulli and a Gamma distribution. In the case of a Bernoulli distribution, we assume that a loss $\hat{x} > 0$ occurs with probability $p \in (0,1)$, and no loss with probability $1-p$, i.e. $X \sim \text{Ber}(\hat{x}, p)$. For HARA utility we assume that $\hat{x} \leq w$, for logarithmic utility $\hat{x} < w$. For a Gamma distribution with parameters $\xi, \mu > 0$ and density

$$f_{\xi,\mu}(x) = \frac{\mu^\xi}{\Gamma(\xi)}\, x^{\xi-1}\, e^{-\mu x}\, \mathbb{1}_{(0,\infty)}(x), \quad x \in \mathbb{R},$$

we use the notation $\Gamma(\xi, \mu)$. Note that $\Gamma(\cdot)$ denotes the ordinary gamma function. The Gamma distribution with unbounded support will only be considered in the case of CARA utility.

Remark 4.3. *The following result is a simple consequence of Föllmer & Schied (2011), Proposition 2.39: Let u : dom $u \to \mathbb{R}$ be a Bernoulli utility function. We assume that $\mathbb{R}_+ \subseteq$ dom u, $X \leq w$ and $\pi \leq w$. Then the following assertions hold:*

(a) We have $\nu^*(\pi) = 1$ if $\pi \leq \mathbb{E}[X]$, and $\nu^*(\pi) > 0$ if $\pi \leq w - c_X$, where c_X is the certainty equivalent *given by the equation $\mathbb{E}[u(X)] = u(c_X)$.*

(b) If u is differentiable, then

$$\nu^*(\pi) = 1 \;\Leftrightarrow\; \pi \leq \mathbb{E}[X]$$

and

$$\nu^*(\pi) = 0 \ \Leftrightarrow \ \pi \geq w - \frac{\mathbb{E}[(w-X)u'(w-X)]}{\mathbb{E}[u'(w-X)]}.$$

If $\pi < w - \frac{\mathbb{E}[(w-X)u'(w-X)]}{\mathbb{E}[u'(w-X)]}$, *then* $\nu^*(\pi) > 0$.

A risk-averse buyer purchases full insurance, if and only if the premium does not exceed the expected loss. Insurers will, however, always charge premiums that are larger in order to avoid ruin. In this case, full insurance is never optimal.
For the special case of Bernoulli-distributed random variables Schrinner (1997) discussed

$$\nu^*(\pi) \begin{cases} = 1, & \text{if } \pi \leq \mathbb{E}[X], \\ < 1, & \text{if } \pi > \mathbb{E}[X], \end{cases}$$

in the context of premium tax. He argued that higher premium taxes lead to higher premiums and therefore to a larger deviation of the premium from the expected loss, which results in less demand for insurance.

Theorem 4.4. *The solutions to Problem 4.2 for specific utility functions and loss distributions are as follows:*

(i) **CARA-utility**: *Consider the Bernoulli utility* $u(x) = u_1^\kappa(x), \kappa > 0$.

- *Assume that losses are Bernoulli-distributed, i.e.* $X \sim Ber(\hat{x}, p)$. *Then the optimal insurance contract is characterized by*

$$\nu^*(\pi) = \begin{cases} 0, & \pi \ \geq \ \frac{p\hat{x}e^{\kappa\hat{x}}}{1-p+pe^{\kappa\hat{x}}}, \\ 1 - \frac{1}{\kappa\hat{x}} \ln\left(\frac{\frac{1}{p}-1}{\frac{\hat{x}}{\pi}-1}\right), & \frac{p\hat{x}e^{\kappa\hat{x}}}{1-p+pe^{\kappa\hat{x}}} \ > \ \pi \ > \ p\hat{x}, \\ 1, & p\hat{x} \ \geq \ \pi. \end{cases}$$

- *Assume that losses are Gamma-distributed, i.e.* $X \sim \Gamma(\xi, \mu)$, *and assume that* $\kappa < \mu$. *Then the optimal insurance contract is given by*

$$\nu^*(\pi) = \begin{cases} 0, & \pi \ \geq \ \frac{\xi}{\mu-\kappa}, \\ 1 + \frac{\xi}{\pi\kappa} - \frac{\mu}{\kappa}, & \frac{\xi}{\mu-\kappa} \ > \ \pi \ > \ \frac{\xi}{\mu}, \\ 1, & \frac{\xi}{\mu} \ \geq \ \pi. \end{cases}$$

(ii) **HARA-utility**: *Consider the Bernoulli utility* $u(x) = u_2^\lambda(x)$, $\lambda \in (0,1)$. *We set* $\zeta = \frac{1}{1-\lambda}$.

- *Assume that losses are Bernoulli-distributed, i.e.* $X \sim Ber(\hat{x}, p)$. *We suppose that* $0 < \hat{x} \leq w$. *Then the optimal insurance contract is*

$$\nu^*(\pi) = \begin{cases} 0, & \pi \ \geq \ \frac{p\hat{x}w^{1-\lambda}}{pw^{1-\lambda}+(1-p)(w-\hat{x})^{1-\lambda}}, \\ \frac{\pi^\zeta(1-p)^\zeta(\hat{x}-w)+p^\zeta(\hat{x}-\pi)^\zeta w}{\pi^\zeta(1-p)^\zeta(\hat{x}-\pi)+p^\zeta(\hat{x}-\pi)^\zeta \pi}, & \frac{p\hat{x}w^{1-\lambda}}{pw^{1-\lambda}+(1-p)(w-\hat{x})^{1-\lambda}} \ > \ \pi \ > \ p\hat{x}, \\ 1, & p\hat{x} \ \geq \ \pi. \end{cases}$$

(iii) **Logarithmic utility**: *Consider the logarithmic utility* $u_2^0(x) = \log(x)$, *i.e. the limiting case of HARA-utility for* $\lambda = 0$.

- *Assume that losses are Bernoulli-distributed, i.e.* $X \sim Ber(\hat{x}, p)$. *We suppose that* $0 < \hat{x} < w$. *Then the optimal insurance contract is characterized by*

$$\nu^*(\pi) = \begin{cases} 0, & \pi \ \geq \ \frac{p\hat{x}w}{w+\hat{x}(p-1)}, \\ \frac{\pi(w-\hat{x})-p\hat{x}(w-\pi)}{\pi(\pi-\hat{x})}, & \frac{p\hat{x}w}{w+\hat{x}(p-1)} \ > \ \pi \ > \ p\hat{x}, \\ 1, & p\hat{x} \ \geq \ \pi. \end{cases}$$

Proof. See Section A. □

In order to gauge the effect of a modified tax on insurance demand, we now compute the modification of the effective premiums for different tax systems.

Lemma 4.5. *We denote the taxed premium in a system with premium tax by $\bar{\pi}_{PT}$. Assume now the counterfactual situation that VAT and premium tax can be deducted from each other and that at the same time all tax savings are passed to a corporate buyer of insurance. In this case, the effective premium equals*

$$\bar{\pi}_{VAT} = \gamma \cdot \bar{\pi}_{PT}, \qquad \gamma := 1 - \frac{\tau_{VAT}\alpha + \tau_{PT}}{1 + \tau_{PT}},$$

where α denotes the input ratio.

Proof. The result follows from $\bar{\pi}_{VAT} = \bar{\pi}_{PT} - E$ where tax savings E are computed according to eq. (3) with $\bar{\pi}_{PT} = (1 + \tau_{PT})\beta U$. □

Example 4.6. *For an input ratio $\alpha =$ 5.2% and $\tau_{VAT} = \tau_{PT} =$19%, we obtain $\gamma \approx 0.832$, i.e. the effective premium reduces to 83.2% of the original premium, if deduction is permitted.*

Before we can analyze the impact of alternative tax systems on demand, we need to specify how premiums are calculated net of taxes. We focus on two examples of classical premium principles, namely the expected value principle and the standard deviation principle, see e.g. Chapter 12 in Schmidt (2009). We also investigated the semi-standard deviation principle which leads to similar results as the standard deviation principle; for this reason it is not included in the case studies below. However, we provide the corresponding formulas. Untaxed premiums with safety loading $\delta > 0$ are given in Table 2 for the considered loss distributions. Note that $\underline{\Gamma}(\cdot,\cdot)$ denotes the upper incomplete gamma function. Adjusting tax payments, optimal insurance contracts can finally be computed according to Theorem 4.4.

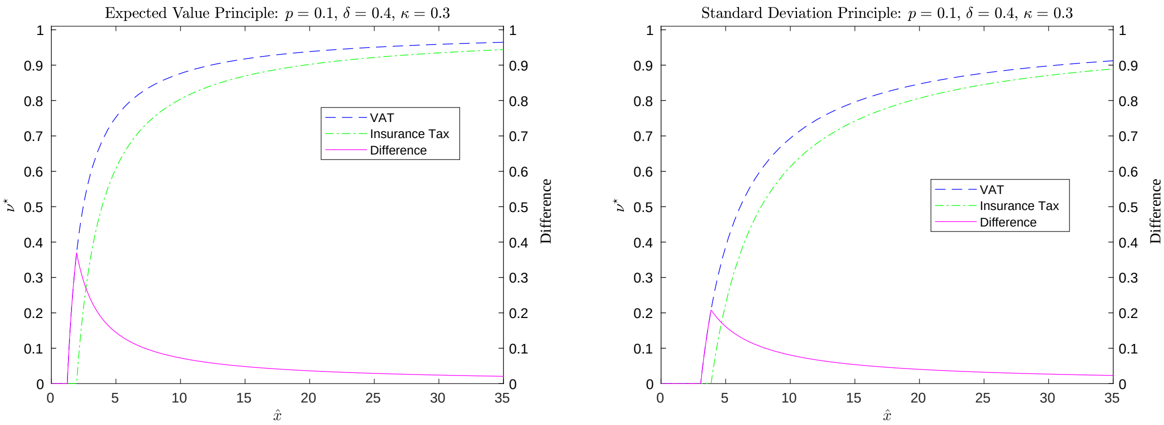
Premium Principle	$\boldsymbol{X \sim \mathrm{Ber}(\hat{x}, p)}$	$\boldsymbol{X \sim \Gamma(\xi, \mu)}$
Expected Value Principle	$p\hat{x}(1+\delta)$	$\frac{\xi}{\mu}(1+\delta)$
Standard Deviation Principle	$p\hat{x}\left(1+\delta\sqrt{\frac{1-p}{p}}\right)$	$\frac{\xi}{\mu}\left(1+\delta\frac{1}{\sqrt{\xi}}\right)$
Semi-Standard Deviation Principle	$p\hat{x}\left(1+\delta\frac{1-p}{\sqrt{p}}\right)$	$\frac{\xi}{\mu}\left(1+\delta\frac{1}{\xi}\sqrt{\frac{1}{\Gamma(\xi)}(\xi^{\xi}\,e^{-\xi}+\xi\,\underline{\Gamma}(\xi,\xi))}\right)$

Table 2: Computation of untaxed premiums.

The following examples analyze the impact of different tax systems on insurance demand. In all case studies, we assume $\tau_{PT} = 19\%$ and $\gamma = 0.832$ according to Example 4.6.

Example 4.7. *In the first numerical example, we consider losses $X \sim Ber(\hat{x}, p)$ and a policyholder with CARA-utility $u(x) = u_1^{\kappa}(x), \kappa > 0$. We choose $p = 0.1$ and vary $\hat{x}$. Risk aversion is set to $\kappa = 0.3$, and the safety loading equals $\delta = 0.4$.*

Figure 1 displays optimal insurance contracts ν^* for the two different tax systems. In the case of CARA-utility, these do not depend on the initial endowment of the policyholder. As expected, if deduction is permitted, the demand for insurance is increased. The difference in demand initially increases for small loss sizes $\hat{x}$ and decreases towards a small level for larger loss sizes. Comparing Figures (a) and (b), we observe similar shapes of the functions for both premium principles. Due to higher premiums for the standard deviation principle, the optimal demand for insurance is smaller than in the case of the expected value principle.

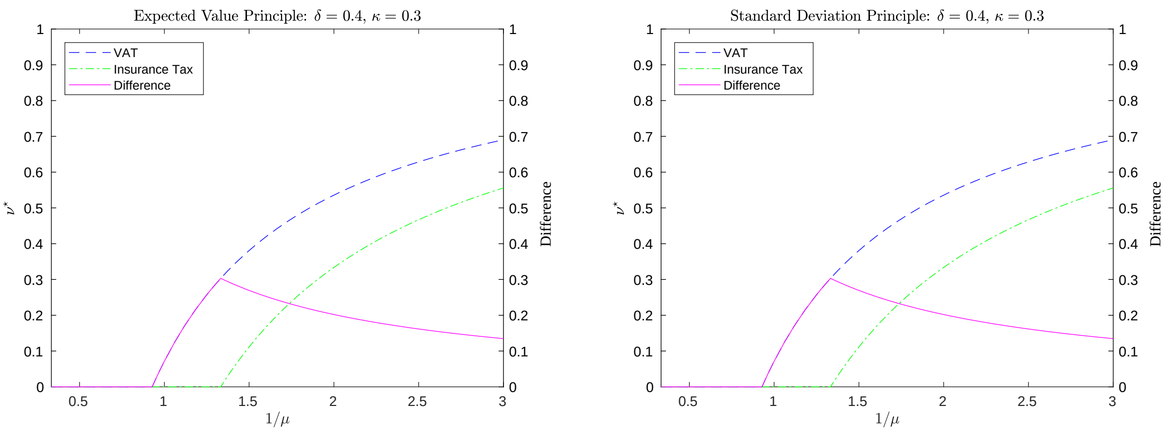


(a) (b)

Figure 1: Insurance demand in Example 4.7 for expected value and standard deviation principle.

Example 4.8. *In the second numerical example, we consider losses $X \sim \Gamma(\xi, \mu)$ and a policyholder with CARA-utility $u(x) = u_1^\kappa(x), \kappa > 0$. We choose $\xi = 1$ and vary $1/\mu$. Risk aversion is again set to $\kappa = 0.3$, and the safety loading equals $\delta = 0.4$.*

Figure 2 displays optimal insurance contracts ν^* for the two different tax systems. Again, if deduction is permitted, the demand for insurance is increased. The difference in demand is zero for small expected loss sizes $1/\mu \leq 0.92$, increases for $1/\mu \in (0.92, 1.33)$, and decreases for larger losses.



(a) (b)

Figure 2: Insurance demand in Example 4.8 for expected value and standard deviation principle.

Example 4.9. *In the third numerical example, we consider losses $X \sim Ber(\hat{x}, p)$ and a policyholder with HARA-utility $u(x) = u_2^\lambda(x), \lambda \in (0, 1)$. We choose $p = 0.1$ and vary $\hat{x}$. We set $\lambda = 0.2$. The safety loading equals $\delta = 0.01$. Initial wealth is $w = 300$.*

Figure 3 displays optimal insurance contracts ν^* for the two different tax systems. Again, we obtain that the demand for insurance is increased, if deduction is permitted. The difference in demand is large for small loss sizes $\hat{x}$ and decreases for larger losses. It remains larger than 0.2 for all values of $\hat{x}$ in the case of the expected value principle resp. for all values of $\hat{x} \geq 10.2$ in the case of the standard deviation principle.

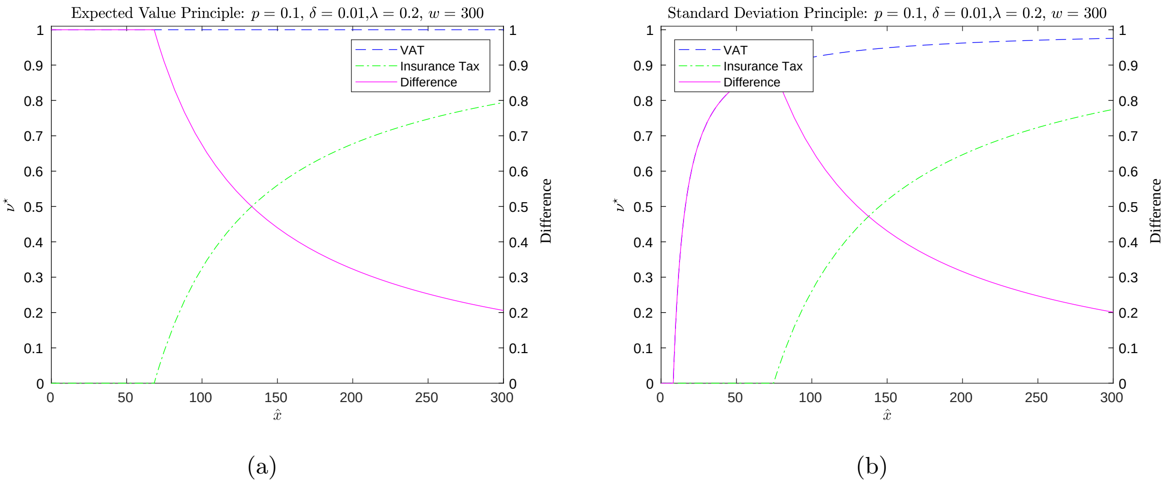


Figure 3: Insurance demand in Example 4.9 for expected value and standard deviation principle.

Example 4.10. *Finally, we consider the same situation as in Example 4.9, but keep* $\hat{x} = 280$ *fixed and vary* λ*. Risk aversion decreases with increasing* λ*. Figure 4 displays optimal insurance contracts* ν^* *for the two different tax systems. With VAT, insurance demand stays close to 1 for small values of* λ*. Premium tax leads to a higher cost of insurance, and insurance demand is significantly lower. In the case of premium tax, insurance demand decreases to 0 as risk aversion goes to* 0*, i.e.* λ *approaches* 1*. In the case of VAT, this effect occurs only if premiums are computed according to the standard deviation principle.*

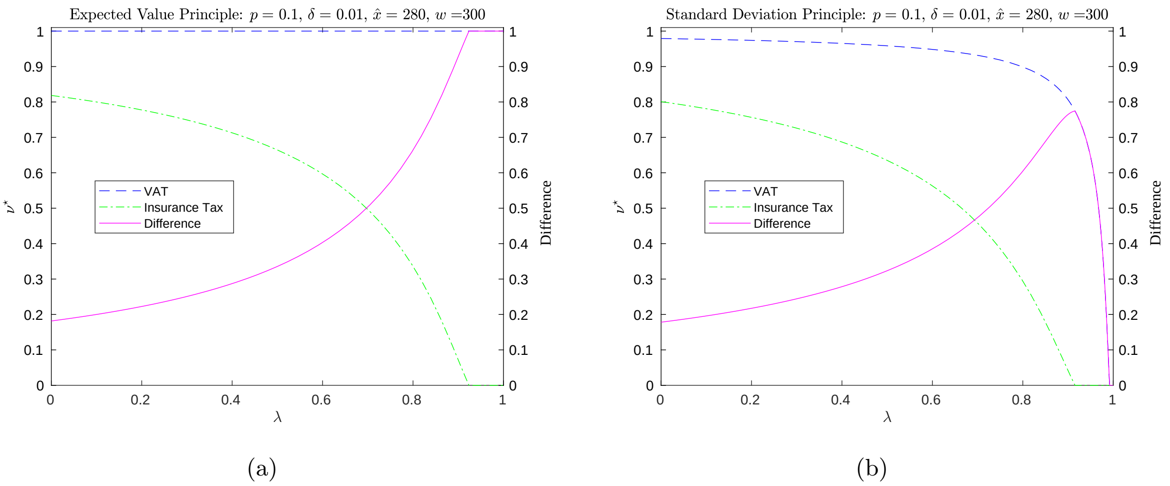


Figure 4: Insurance demand in Example 4.10 for expected value and standard deviation principle.

In summary, our case studies show that the tax system may have a substantial impact on insurance demand of corporate buyers of insurance. The size of this effect depends on the loss distribution and the utility of the policyholder, in particular on the size of potential losses and the risk aversion of the policyholder.

5 Impact on Competitiveness

The current tax system in Germany with premium tax does not allow that premium tax and VAT are deducted from each other. If such deductions were permitted, as, for example, in an hypothetical tax system that charges VAT on insurance contracts (instead of premium tax), overall tax payments would be reduced.

Consider a corporate insurance holder. We compare two tax systems: a realistic tax system with premium tax, and a counterfactual tax system that permits the full deduction of VAT and premium tax from each other. We assume that the resulting tax savings lead to a reduction of the sales prices of the corporate policyholder. The reduced prices increase the relative competitiveness of a domestic firm that benefits from a modified tax system in contrast to its international competitors.

We design a stylized model that captures this effect and allows its quantification. There are two firms that produce different goods $i = 1, 2$ that they sell for prices p_i, $i = 1, 2$. We assume that demand for the two goods in the economy is the solution to a utility maximization of a representative consumer.

Problem 5.1 (Utility Maximization)**.** *The utility function of the representative consumer with budget $w \in \mathbb{R}_+$ is denoted by $u : \mathcal{X} \to \mathbb{R}$, $\mathcal{X} = \mathbb{R}_+ \times \mathbb{R}_+$. The consumer's demand $x^* = (x_1^*, x_2^*)$ solves her utility maximization problem*

$$x^* \in \operatorname{argmax}_{x \in \mathbb{R}_+^2} u(x_1, x_2)$$

subject to her budget constraint

$$p_1 x_1 + p_2 x_2 = w.$$

The following preliminary lemma computes the price reduction when the tax system is changed.

Lemma 5.2. *Let α be the rate of input of insurance companies, and let β be the insurance ratio of company 1 as defined in eq.* (2)*. If company* 1 *is a domestic company, a modification of the domestic tax system, as described in Lemma 2.4, decreases the price of its product by $\theta_{\alpha,\beta} \cdot p_1$, where*

$$\theta_{\alpha,\beta} := (\tau_{VAT}\alpha + \tau_{PT})\,\beta,$$

and p_1 denotes the original price with premium tax. I.e. the unit price decreases to $\tilde{p}_1 = (1 - \theta_{\alpha,\beta}) \cdot p_1$.

Proof. We have $\theta_{\alpha,\beta} = \frac{E}{U}$ where U is the revenue or business volume of the company. Now, the result follows from eq. (3). □

In two case studies, we will now illustrate how a modification of the tax system may change product demand. In the first example, the representative consumer has a utility function of Cobb-Douglas type, in the second with constant elasticity of substitution.

5.1 Cobb-Douglas Utility Function

We recall that a *Cobb-Douglas Utility Function* has the form

$$u_a(x_1, x_2) := x_1^a x_2^{1-a}, \qquad a \in (0, 1).$$

Solving the optimization problem 5.1, one obtains the solution

$$x_1^{(a)} = \frac{aw}{p_1}, \qquad x_2^{(a)} = \frac{(1-a)w}{p_2}.$$

We compare the change in competitiveness of a domestic and a foreign firm, if the domestic system is changed as described before. For this purpose, we assume that company 1 is domestic and company 2 foreign. The price of the product of company 2 is p_2 and fixed, but the price of the product of company 1 is a function of the domestic tax system.

Lemma 5.3. *Let p_1 be the original price of product 1 and $x_1^{(a)}$ the corresponding demand. Suppose that $\tilde{p}_1$ is the price of product 1 after modifying the tax system, see Lemma 5.2, and $\tilde{x}_1^{(a)}$ the corresponding demand. Setting $\Delta x_1^{(a)} = \tilde{x}_1^{(a)} - x_1^{(a)}$, we obtain:*

$$\frac{\Delta x_1^{(a)}}{x_1^{(a)}} = \frac{\theta_{\alpha,\beta}}{1-\theta_{\alpha,\beta}}.$$

Proof. This is an application of Lemma 5.2 to the solution of the optimization problem. □

The relative shift in demand does neither depend on the available budget w nor the preference parameter a.

Example 5.4. *Taking the numbers from Example 2.5, we compute $\theta_{\alpha,\beta} \approx 0.1\%$, thus*

$$\frac{\Delta x_1^{(a)}}{x_1^{(a)}} \approx 0.1\%, \quad \forall a \in (0,1).$$

The price of the product of the domestic company and its competitiveness is almost not affected by a modification of the tax system. The reason is that the insurance ratio of companies is typically small. In addition, the rate of input of insurance companies is not very large.

Example 5.5. *Insurance contracts are an input to the production of goods. Their contribution varies across different industry sectors and so does the effect of a modification of the tax system on production costs and product prices. Suppose that the input ratio is set to α =5.2% as in the previous example. Insurance ratios for different US industry sectors are based on a survey of MarketStance and were obtained from sigma (2012) (p. 17). The data are displayed in Table 3. Again, $\theta_{\alpha,\beta}$ and $\frac{\Delta x_1^{(a)}}{x_1^{(a)}}$ are computed according to our model. In all cases, the effects are very small.*

Industrial Sector	**Premium/Business Vol.** β in %	**Saving/Business Vol.** $\theta_{\alpha,\beta}$ in %	**Shift in Demand** $\Delta x_1^{(a)}/x_1^{(a)}$ in %
Mining	0.80	0.16	0.16
Construction	1.31	0.26	0.26
Manufacturing	0.31	0.06	0.06
Transport, communication, utilities	1.21	0.24	0.24
Retail trade	0.36	0.07	0.07
Wholesale trade	0.14	0.03	0.03
Financial	0.38	0.08	0.08
Services	0.70	0.14	0.14

Table 3: Shift in product demand related to industrial sectors.

5.2 Constant Elasticity of Substitution Utility Function

We recall the definition of a utility function with *constant elasticity of substitution (CES)*:

$$u_{a,b}(x_1,x_2) := \left(a x_1^b + (1-a) x_2^b\right)^{\frac{1}{b}},$$

where $a \in (0,1)$ and $b \neq 0$. The latter quantity is called the *parameter of substitution*. Again, we denote the budget of the consumer by w. The consumer's optimal demand is

$$x_1^{a,b} = \frac{w\ (p_1/a)^{-\eta}}{a^\eta p_1^{1-\eta} + (1-a)^\eta p_2^{1-\eta}}, \qquad x_2^{a,b} = \frac{w\ (p_2/(1-a))^{-\eta}}{a^\eta p_1^{1-\eta} + (1-a)^\eta p_2^{1-\eta}},$$

where $\eta := \frac{1}{1-b}$ denotes the elasticity of substitution.

Lemma 5.6 (Shift in Demand)**.** *Let p_1 be the original price of product 1 and $x_1^{a,b}$ the corresponding demand. Suppose that $\tilde{p}_1$ is the price of product 1 after modifying the tax system, see Lemma 5.2, and $\tilde{x}_1^{a,b}$ the corresponding demand. Setting $\Delta x_1^{a,b} = \tilde{x}_1^{a,b} - x_1^{a,b}$, we obtain:*

$$\frac{\Delta x_1^{a,b}}{x_1^{a,b}} = (1-\theta_{\alpha,\beta})^{-\eta} \frac{a^\eta \; p_1^{1-\eta} + (1-a)^\eta \; p_2^{1-\eta}}{(1-\theta_{\alpha,\beta})^{1-\eta} \; a^\eta \; p_1^{1-\eta} + (1-a)^\eta \; p_2^{1-\eta}} - 1.$$

Proof. This is an application of Lemma 5.2 to the solution of the optimization problem. □

In contrast to a Cobb-Douglas utility, the relative demand shift depends on the parameters of the utility and the price level of the products in the case of CES-utility. The impact of these inputs on demand is illustrated in Figure 5 for the parameter values given in Table 4.

Saving:	$\theta_{\alpha,\beta} = 0.1\%$
Share Parameter:	$a \in (0, 1)$
Price of Product 1:	$p_1 = 1$
Price of Product 2:	$p_2 \in [0, 2]$

Table 4: Parameters for case studies with CES utility function.

In particular, we consider different parameters of substitution b and elasticity of substitution $\eta = 1/(1-b)$. For $\eta > 1$ the products are gross substitutes, for $\eta < 1$ they are gross complements. We fix $p_1 = 1$ and vary η, a and p_2. The resulting relative demand shifts are displayed in Figure 5. In case of gross substitutes ($\eta > 1$), the increase in demand caused by the price change is, of course, higher than in case of gross complements. The effect is, however, in all cases very small.

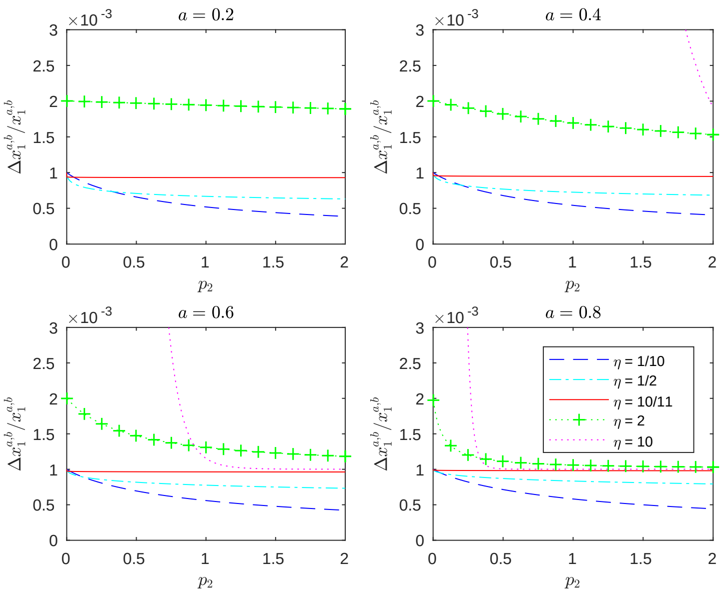


Figure 5: Relative shift in demand with CES utility functions.

All case studies clearly indicate that the international competitiveness (in terms of product pricing) of corporate policyholders is almost not affected by the difference of premium tax and VAT.

6 Impact on Ruin Probability

In this section we investigate the impact of the premium tax on the ruin probability of insurance companies. For this purpose, we extend the classical Cramér-Lundberg model by including taxes and compare ruin probabilities for different systems of taxation. A concise introduction to ruin theory is Mikosch (2009). A comprehensive presentation can be found in Asmussen & Albrecher (2010).

Let $(\Omega, \mathcal{F}, P)$ be a probability space. We consider a family of *risk processes* of insurance companies $(R_t^w)_{t\geq 0}$ enumerated by the initial wealth $R_0^w = w \in \mathbb{R}$. The ruin probability of these companies is a function of initial wealth:

$$\psi^w(\pi) := P\left(\inf_{t\geq 0} R_t^w < 0\right).$$

For later reference, we also emphasize the dependence on the premium rate.

6.1 The Cramér-Lundberg Model

In the current section we recall the classical Cramér-Lundberg model and its basic definition. On the basis of Mikosch (2009) and Asmussen & Albrecher (2010), we collect the results that will be needed for an analysis of the impact of insurance tax on ruin.

Model 6.1 (Cramér-Lundberg)**.** *Denote the initial capital of the insurance company by $w \in \mathbb{R}$ and its premium rate by $\pi \in \mathbb{R}$. Insurance losses are modeled by a compound Poisson process $\left(\sum_{k=1}^{N_t} X_k\right)_{t\geq 0}$ where individual losses $(X_k)_{k\in\mathbb{N}}$ are strictly positive, integrable, identically distributed with law B, jointly independent and independent of the Poisson process $(N_t)_{t\geq 0}$ with intensity $\vartheta > 0$. The risk process in the Cramér-Lundberg model is given by*

$$R_t^w = w + \pi t - \sum_{k=1}^{N_t} X_k.$$

Wald's equation and the strong law of large numbers imply that $\frac{1}{t}\sum_{k=1}^{N_t} X_k \xrightarrow[t\to\infty]{} \vartheta\,\mathbb{E}[X_1] =: r$ almost surely. It is well-known that ruin occurs with probability 1, unless the net profit condition (NPC) holds, i.e. $\pi > r$. This is equivalent to premium payments being larger than the expected value of the losses for any time horizon t, i.e.

$$\pi t \;=\; (1+\rho)\,\mathbb{E}\left[\sum_{k=1}^{N_t} X_k\right]$$

with safety loading $\rho = \frac{\pi - r}{r} > 0$. If the NPC holds, the asymptotic behaviour of the ruin probability ψ^w for $w \to \infty$ can be characterized; large w corresponds to high initial capital. We recall the key results for light-tailed and heavy-tailed losses in the Cramér-Lundberg model.

Notation. If $\lim_{w\uparrow\infty} \frac{\psi^w(\pi)}{\varphi^w(\pi)} = 1$, we write $\psi^w(\pi) \sim \varphi^w(\pi)$.

The classical result of ruin theory considers the case of light-tailed distributions and involves the Cramér-Lundberg coefficient. Assume that the moment-generating function of X_1, i.e. $\hat{B}(h) = \int e^{hz} dB(z) = \mathbb{E}\left[e^{hX_1}\right]$, exists for all $h \in (-h_0, h_0)$ for some $h_0 > 0$. The Cramér-Lundberg coefficient $l > 0$, if it exists, is the unique solution of the equation

$$\hat{B}(l) = 1 + \frac{\pi l}{\vartheta}.$$

Theorem 6.2 (Cramér-Lundberg Approximation)**.** *Assume that the NPC holds and that the distribution of* X_1 *has a density and a moment-generating function in some neighborhood of* 0*. In addition, we suppose that the Cramér-Lundberg coefficient* $l > 0$ *exists. Setting* $C = \frac{\pi - r}{\vartheta \hat{B}'(l) - \pi}$*, the asymptotic behaviour of the ruin probability can be characterized as follows:*

$$\lim_{w \to \infty} e^{lw} \psi^w(\pi) = C,$$

i.e. $\psi^w(\pi) \sim Ce^{-lw}$ *as* $w \to \infty$*.*

Example 6.3. *For independent, exponentially distributed losses* $X_1, X_2, \ldots$ *with parameter* $\iota > 0$*, i.e.* $X_k \sim Exp(\iota)$*,* $k \in \mathbb{N}$*, the Cramér-Lundberg coefficient is* $l = \iota - \frac{\vartheta}{\pi}$*, and the ruin probability equals the asymptotic approximation of Theorem 6.2:*

$$\psi^w(\pi) = \frac{\vartheta}{\iota\pi} e^{-\left(\iota - \frac{\vartheta}{\pi}\right)w}. \tag{4}$$

So far, we considered light-tailed losses. In the case of heavy-tailed losses, the occurrence of ruin is qualitatively different from the light-tailed case. For light-tailed loss distributions, ruin happens if a large number of sufficiently large claims accumulate. For heavy-tailed loss distributions, ruin can occur spontaneously and is typically due to a large single claim. Quantitatively, this is related to the integrated tail distribution. The corresponding theorem of Embrechts & Veraverbeke (1982) requires the notion of subexponential distributions.

Remark 6.4. *(a) If the positive random variable* X *has distribution function* F*, then the function* $F_{X,I} : \mathbb{R} \to [0, 1]$ *with*

$$F_{X,I}(x) = \left(\frac{1}{\mathbb{E}[X]} \int_0^x (1 - F(y))\, dy \right) \cdot \mathbb{1}_{(0,\infty)}(x)$$

is the integrated tail distribution function *of* X*. The function* $F_{X,I}$ *is a distribution function of a probability measure on the positive half line.*

(b) Subexponential distributions provide a natural definition of being heavy-tailed. They formalize that the tail of the sum $S_n = X_1 + \cdots + X_n$ *is essentially determined by the tail of the maximum* $M_n = \max_{k=1,\ldots,n} X_k$ *for independent copies of the distribution of* X_1*. A formal definition is*

$$\forall n \geq 2: \quad \lim_{x \to \infty} \frac{P(S_n > x)}{P(X_1 > x)} = n.$$

Theorem 6.5. *Assume that the NPC holds. In addition, suppose that the losses* X_k *have a density and that* $F_{X_1,I}$*, the integrated tail distribution function of* X_1*, is subexponential. Then*

$$\lim_{w \to \infty} \frac{\psi^w(\pi)}{1 - F_{X_1,I}(w)} = \frac{1}{\rho} = \frac{r}{\pi - r},$$

i.e. $\psi^w(\pi) \sim \frac{1 - F_{X_1,I}(w)}{\rho}$ *as* $w \to \infty$*.*

Example 6.6. *Examples of parametric distributions that satisfy the conditions of this theorem can be found in Table 3.2.19 in Mikosch (2009). These include the log-normal and the Pareto distribution.*

6.2 The Cramér-Lundberg Model with Taxes

We extend the model and add taxes.

Model 6.7 (Cramér-Lundberg Model with Taxes)**.** *Let $\tau \in [0,1]$ be a constant tax rate that is charged on the gross premium income $\pi \in \mathbb{R}$. We denote taxed premiums by $\bar{\pi} = (1+\tau)\cdot\pi$. We assume that the* realized tax *charged on the insurer's input costs $L \in \mathbb{R}$ is given by the tax rate $\varepsilon\tau$ which we represent as a fraction $\varepsilon \in [0,1]$ of the premium tax rate $\tau \in [0,1]$. The term* realized tax *refers to the tax on inputs minus deductions that are allowed. The costs L do not include insurance payments due to losses. The* after-tax risk process *is*

$$R_t^{w,\tau,\varepsilon} = w + \left[\frac{\bar{\pi}}{1+\tau} - (1+\varepsilon\tau)L\right] t - \sum_{k=1}^{N_t} X_k. \tag{5}$$

The effective insurance premium (after subtracting all expenses and taxes) is

$$\pi^{\tau,\varepsilon} := \frac{\bar{\pi}}{1+\tau} - (1+\varepsilon\tau)L. \tag{6}$$

The model allows to mimic different tax systems.

(i) For $\tau = \tau_{PT} = \tau_{VAT}$ and $\varepsilon = 1$ we obtain a tax system with premium tax in which VAT is applied to inputs but cannot be deducted from the premium tax payments. This captures the current German tax system, if we choose $\tau = 19\%$.

(ii) For $\tau = \tau_{VAT}$ and $\varepsilon = 0$ we obtain a counterfactual tax system in which premium tax is replaced by VAT. In this case, VAT paid on inputs is fully deductable from VAT paid on insurance premiums.

(iii) Suppose that we are given a tax system with premium tax and VAT as in (i), i.e. $\tau = \tau_{PT} = \tau_{VAT}$ and $\varepsilon = 1$. As in Example 2.2, we consider a counterfactual tax system (ii) in which premium tax is replaced by VAT. We assume that VAT paid on value added of the insurance firm in the new tax system is equal to premium tax revenues in the original tax system. Moreover, we hold π constant. We denote the modified quantities with a tilde. In this case, $\tilde{\tau} = \widetilde{\tau_{VAT}} = \tau_{PT} \cdot \frac{\pi}{\widetilde{W}}$ and $\tilde{\varepsilon} = 0$ where $\widetilde{W}$ is the value that the insurance company adds to its inputs by producing the insurance contract, see Section 2. The modified taxed premium rate is given by $\bar{\pi} = (1+\tilde{\tau})\cdot\pi > (1+\tau_{PT})\cdot\pi$. We implicitly assumed that the higher tax on the premium is payed by the policyholder. The financial situation of the insurance company is thus improved in this case, since it can take advantage of tax deductions.

(iv) Suppose that we are in a counterfactual tax system (ii). We can extend the arguments in (iii) to construct a corresponding tax system with premium tax. We assume that insurance tax revenues in the new tax system are equal to VAT paid on value added of the insurance firm in tax system (ii). Moreover, we hold π constant. In this case, the premium tax rate needs to be adjusted, i.e. $\tilde{\tau} = \widetilde{\tau_{PT}} = \tau_{VAT} \cdot \frac{\widetilde{W}}{\pi}$ and $\tilde{\varepsilon} = 1$. The modified taxed premium rate is given by $\bar{\pi} = (1+\tilde{\tau})\cdot\pi < (1+\tau_{VAT})\cdot\pi$. This describes the opposite scenario to the situation in (iii). The modification of the tax system leads to a higher tax burden of the insurance company, since tax deductions are no longer possible. The benefits are in this case transferred to the policyholders.

The risk process $R^{w,\tau,\varepsilon}$ defined in eq. (5) is a function of the tax parameters τ and ε. We investigate how ruin probabilities depend on the tax system. In contrast to the examples (iii) and (iv) above, we now keep $\bar{\pi}$ fixed instead of π. Tax expenses and tax savings are not transferred to the buyer of insurance, but are fully absorbed by the insurance firm. This implies, in particular, that the design of the tax system and a modified tax rate on premiums alter the financial resources and ruin probability of the insurance company.

The effective insurance premium is computed according to eq. (6). For both the light-tailed and the heavy-tailed case we consider the dependence of the ratio $\psi^w(\pi^{\tau,\varepsilon})/\psi^w(\pi^{0,0})$ on the tax rate τ and compare the cases $\varepsilon = 1$ and $\varepsilon = 0$ that correspond to a system with premium tax and VAT, respectively.

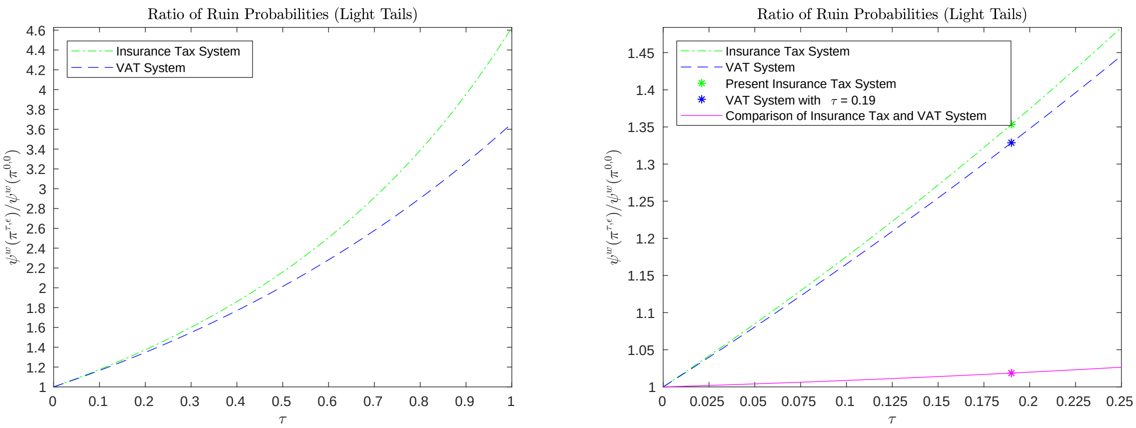
Example 6.8. *First, we consider light-tailed loss distributions. If the conditions of Theorem 6.2 hold, then clearly* $\frac{\psi^w(\pi^{\tau,\varepsilon})}{\psi^w(\pi^{0,0})} \sim \frac{C(\pi^{\tau,\varepsilon})e^{-l(\pi^{\tau,\varepsilon})w}}{C(\pi^{0,0})e^{-l(\pi^{0,0})w}}$ *with* $C(\pi) = \frac{\pi - r}{\vartheta \hat{B}'(l(\pi)) - \pi}$ *and Cramér-Lundberg coefficient* $l(\pi)$ *for any effective insurance premium* π*. In the case of exponential losses, the approximation equals the exact ruin probability. Let* $X_1, X_2, \dots$ *be exponentially distributed with parameter* $\iota > 0$*. Then*

$$\frac{\psi^w(\pi^{\tau,\varepsilon})}{\psi^w(\pi^{0,0})} = \frac{\pi^{0,0}}{\pi^{\tau,\varepsilon}} e^{\vartheta w\left(\frac{1}{\pi^{\tau,\varepsilon}} - \frac{1}{\pi^{0,0}}\right)} \tag{7}$$

In the numerical example, we choose $\iota = 1$ *and* $\vartheta = 1$*, thus* $r = 1$*. Since the NPC should be satisfied, we assume* $\pi^{0,0} = 2$*. We set*[9] $w = 1 \approx 0.87 = 43.5\% \cdot \pi^{0,0}$ *and choose* $L = \alpha\pi^{0,0} = 2\alpha$*. Observe that* $\bar{\pi} = \pi^{0,0} + L$*. Setting the mean rate of input* $\alpha = 5.2\%$ *as in Example 2.1, we obtain* $L = 0.104$ *and deduce from formula* (6)*:*

$$\pi^{\tau,\varepsilon} := \frac{2+L}{1+\tau} - (1+\varepsilon\tau)L = \frac{2.104}{1+\tau} - 0.104 \cdot (1+\varepsilon\tau) \approx \frac{2.1}{1+\tau} - \frac{1+\varepsilon\tau}{10}.$$

Plugging this result into eq. (7)*, we compute the ratio of ruin probabilities as a function of* τ*. This is displayed in Figure 6.*



(a) (b)

Figure 6: Ratio of ruin probabilites for exponentially distributed losses.

Clearly, the higher the tax rate τ *the higher the probability of ruin compared to a system without taxes. As expected, the increase of ruin probabilities is stronger in a tax system with premium tax. A change of the tax system from type (i) to type (ii) would thus reduce the probability of ruin. We observe that* $\psi^w(\pi^{0.19,1})/\psi^w(\pi^{0,0}) = 1.353$*,* $\psi^w(\pi^{0.19,0})/\psi^w(\pi^{\tau,0}) = 1.329$*, thus* $\psi^w(\pi^{0.19,1})/\psi^w(\pi^{0.19,0}) = 1.018$*. In Germany, at the prevailing rate of 19%, the ruin probability in a tax system with premium tax is only about 2% larger than the ruin probability in a tax system with VAT.*

Example 6.9. *Second, we consider the heavy-tailed loss distributions. As in the previous example, we assume that*

$$\pi^{\tau,\varepsilon} \approx \frac{2.1}{1+\tau} - \frac{1+\varepsilon\tau}{10},$$

and choose $\vartheta = 1$*. Let* Z *be log-normally distributed with parameters* $(0,1)$*, thus* $\mathbb{E}[Z] = e^{1/2}$*. We assume that the independent losses* $X_1, X_2, \dots$ *have the same distribution as* $e^{-1/2} \cdot Z$*, i.e.* X_k *is*

[9] According to BaFin (2011-2015), Issue 2015, p. 158, Table 520 equity capital of non-life insurance firms in Germany in 2015 was 43.5% of gross premium income.

log-normally distributed with parameters $\left(-\frac{1}{2}, 1\right)$*. This implies* $\mathbb{E}[X_1] = 1$*, thus* $r = 1$ *as in the example of light-tailed losses. We compute:*

$$\frac{\psi^w(\pi^{\tau,\varepsilon})}{\psi^w(\pi^{0,0})} \sim \frac{\pi^{0,0} - 1}{\pi^{\tau,\varepsilon} - 1} = \left(\frac{2.1}{1+\tau} - \frac{11 + \varepsilon\tau}{10}\right)^{-1}$$

This is displayed in Figure 7.

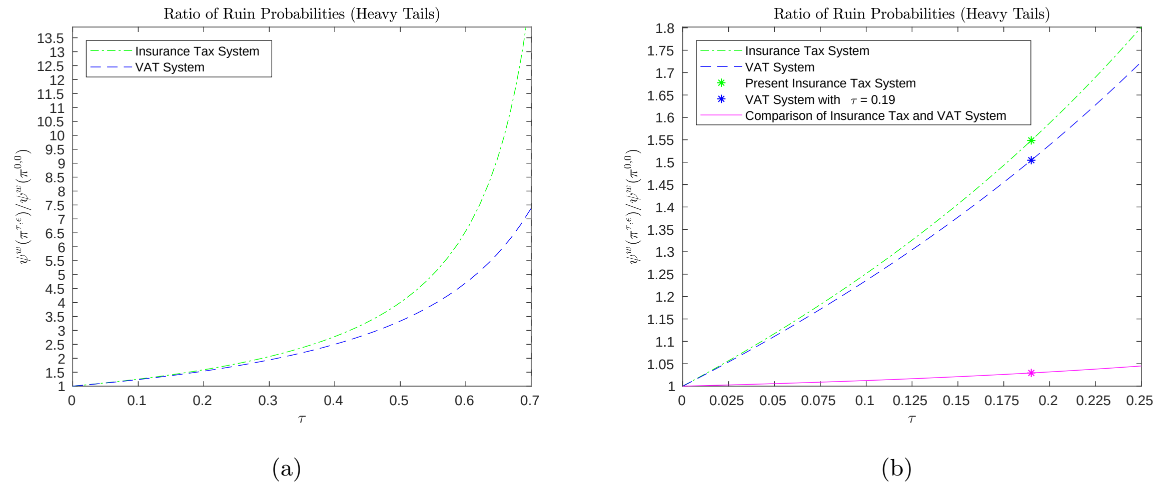


Figure 7: Ratio of ruin probabilites for heavy-tailed loss distributions.

In particular, we obtain $\psi^w(\pi^{0.19,1})/\psi^w(\pi^{0,0}) = 1.549$*,* $\psi^w(\pi^{0.19,0})/\psi^w(\pi^{\tau,0}) = 1.504$*, thus* $\psi^w(\pi^{0.19,1})/\psi^w(\pi^{0.19,0}) = 1.03$*. In Germany, at the prevailing rate of 19%, the ruin probability in a tax system with premium tax is only about 3% larger than the ruin probability in a tax system with VAT. Let us finally stress that the results do not depend on the loss distributions being log-normal; only the condition* $\mathbb{E}[X_1] = 1$ *was used in the derivation.*

The quantities derived in Examples 6.8 & 6.9 and displayed in Figures 6 & 7 are all ratios of ruin probabilities over an infinite time horizon. In absolute terms, annual ruin probabilities of real insurance companies are limited by regulatory standards. Solvency II, for example, restricts annual ruin probabilities to at most 0.5%. Otherwise, companies face serious interventions of the regulator. Our analysis thus indicates that absolute changes of annual ruin probabilities due to a modified tax system would be extremely moderate – on the order of less than 10 basis points. However, even small absolute changes in ruin probabilities might be costly in terms of solvency capital, if regulatory constraints are tight and binding. This issue is discussed in the next section.

Remark 6.10. *Our focus is on premium tax and VAT, and we deduced the implications of tax systems on ruin probabilities from standard results in the literature on ruin theory. The key assumption was that eq.* (6) *describes the tax impact on the risk process. In the context of premium tax and VAT, eq.* (6) *is a reasonable hypothesis. However, the functional dependence of risk processes on other types of taxes might be more complicated than assumed in this paper. We briefly summarize some previous key contributions. Albrecher & Hipp (2007) analyze the effect of tax payments under a loss-carry forward system in the Cramér-Lundberg model. They suppose that taxes are only paid when the company is in a profitable situation, meaning that the risk process is at its running maximum. The authors study ruin probabilities with and without taxes in their model and find that the survival probability with tax is a power of the survival probability without tax, i.e.* $1-\psi_\gamma(w) = (1-\psi_0(w))^{\frac{1}{1-\gamma}}$*, where* $0 < \gamma < 1$ *is the constant tax rate. Moreover, they compute the optimal surplus level at which taxation should start in order to maximize the expected discounted tax payments before ruin. Albrecher, Badescu & Landriault (2008) conduct a* *similar analysis in the dual risk model. The description of the ruin probability with taxes in terms of the ruin probability without taxes becomes more complicated. A considerable generalization of the results is derived by Albrecher, Renaud & Zhou (2008) who embed the model by Albrecher & Hipp (2007) into a general Lévy framework. The relation between ruin probabilities is recovered, and also the structure of many other results is preserved in the Lévy setup. Kyprianou & Zhou (2009) and Albrecher, Borst, Boxma & Resing (2009) introduce a surplus-dependent tax rate.*

7 Impact on Solvency Capital Requirement

As explained in the previous sections, premium tax generates more tax revenues than VAT, if both tax rates are equal. We now compare these two alternative tax systems from the point of view of solvency capital requirements. For this purpose, we assume that insurance firms keep their taxed insurance premiums constant, but retain the tax savings that accrue when premium tax is replaced by VAT. Obviously, the solvency capital requirement is then decreased by this amount, and insurance companies can distribute all tax savings to their shareholders. If this occurs, risk will be back at its original level. In the current section we review the notion of solvency capital requirements in the context of internal models[10] and explain in detail why the dividend payments to shareholders may be increased.

To this end, we review the basic definition of distribution-based monetary risk measures. These include all risk measures that are typically used in practice. For a detailed exposition on the subject we refer to Artzner, Delbaen, Eber & Heath (1999), Föllmer & Schied (2011), and Föllmer & Weber (2015).

Definition 7.1. *Let $(\Omega, \mathcal{F}, P)$ be a probability space, and $\mathcal{X}$ a vector space of random variables on Ω that contains the constants. We identify random variables that are P-almost surely equal. A mapping $\rho : \mathcal{X} \to \mathbb{R}$ is called a* monetary risk measure *on $\mathcal{X}$, if $\rho(X) = \rho(Y)$ for $X = Y$ P-almost surely and if ρ satisfies the following properties:*

(i) Monotonicity*: If $X \geq Y$ P-almost surely, then $\rho(X) \leq \rho(Y)$.*
(Better payoff profiles are less risky.)

(ii) Cash-invariance: *If $m \in \mathbb{R}$, then $\rho(X + m) = \rho(X) - m$.*
(Adding a fixed amount m to the risky position decreases the risk exactly by this amount.)

The risk measure ρ is called distribution-based*, if $\rho(X) = \rho(Y)$ whenever X and Y have the same distribution under P.*

Example 7.2. *Examples of distribution-based monetary risk measures are Value at Risk (V@R) and Average Value at Risk (AV@R), also called expected shortfall, conditional value at risk, tail value at risk, or worst conditional expectation. V@R and AV@R are the basis of the definition of solvency capital requirements in Solvency II and in the Swiss Solvency Test, respectively.*

(i) Value at Risk at level $y \in (0, 1)$ is defined as a quantile:

$$V@R_y(X) := \inf\{m \in \mathbb{R} \mid P(X + m < 0) \leq y\}.$$

It is equal to the smallest monetary amount m that needs to be added to the financial position X such that the probability of a loss does not exceed the level y.

[10] Another approach is the standard formula of Solvency II, a modular construction for the computation of the solvency capital requirement. While an internal model attempts to describe and evaluate the stochastic evolution of the balance sheet of the insurance firm, the standard formula is an auxiliary construction that facilitates the computation of a solvency capital requirement. Aggregation of risk modules is based on correlations and a square-root formula. It is well-known and easily demonstrated that the modular construction cannot be interpreted as an approximation of a capital requirement that limits the probability of ruin to less than 0.5% as requested by Directive 2009/138/EC, see e.g. Pfeifer (2016). In this paper, we focus exclusively on internal models.

(ii) Average Value at Risk at level $y \in (0,1)$ is the average of the V@Rs below y, i.e.

$$AV@R_y(X) := \frac{1}{y} \int_0^y V@R_c(X)\ dc.$$

Under technical conditions, e.g. if X has a continuous distribution, it is equal to the conditional expectation of a loss beyond the $V@R_y(X)$.

We will now explain – in a stylized way – how solvency capital requirements are defined in an internal model. The evolution of assets, liabilities and capital of an insurance firm can be captured by solvency balance sheets at time horizons that are specified by regulators. The time horizon of Solvency II and the Swiss Solvency Test is one year. Table 5 displays the balance sheet of a company at time $t = 0$ and $t = 1$.

$t = 0$

Assets	**Liabilities**
A_0	$E_0 = A_0 - L_0$
	L_0

$t = 1$

Assets	**Liabilities**
A_1	$E_1 = A_1 - L_1$
	L_1

Table 5: Balance sheet of an insurance company for different points in time.

The assets are denoted by A_t, the liabilities by L_t, $t = 0, 1$. The quantities at time $t = 0$ are known, the quantities at time $t = 1$ are random variables. The difference between assets and liabilities $E_t = A_t - L_t$, $t = 0, 1$, is the net asset value (NAV) of the firm. We set $X = E_1 - E_0$ for the change of the NAV over the considered time horizon.

The solvency capital requirement (SCR) for Solvency II is defined in the Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance – Solvency II (see European Commission (2009)):

> The Solvency Capital Requirement should be determined as the economic capital to be held by insurance and reinsurance undertakings in order to ensure that ruin occurs no more often than once in every 200 cases or, alternatively, that those undertakings will still be in a position, with a probability of at least 99.5 %, to meet their obligations to policy holders and beneficiaries over the following 12 months. That economic capital should be calculated on the basis of the true risk profile of those undertakings, taking account of the impact of possible risk-mitigation techniques, as well as diversification effects.

This definition is specified in terms of condition on the acceptability of E_1. An equivalent formulation[11] provides the definition of the SCR under Solvency II:

$$P(E_1 < 0) \leq y \quad \Leftrightarrow \quad \mathrm{V@R}_y(E_1) \leq 0 \quad \Leftrightarrow \quad \mathrm{V@R}_y(E_1 - E_0) \leq E_0 \quad \Leftrightarrow \quad \mathrm{V@R}_y(X) \leq E_0.$$

Setting SCR := $\mathrm{V@R}_y(X)$, the solvency condition of the company becomes

$$\mathrm{SCR} \leq E_0.$$

An analogous argument holds, if V@R is replaced by any other risk measure ρ.[12] The acceptance set of ρ is the family of positions with non-positive risk, i.e.

$$\mathcal{A}_\rho = \{X \in \mathcal{X} :\ \rho(X) \leq 0\}.$$

[11] For simplicity, we assume in this paper that interest rates over the one-year horizon are approximately zero. For adjustments on the definition of the SCR if interest rates are non zero see Christiansen & Niemeyer (2014).

[12] V@R has been criticized in the context of capital regulation, since it neglects losses beyond the V@R and – due to its lack of coherence – it might mislead investment decisions and asset-liability management. In addition, in corporate networks it is possible "to sweep the downside risk under the carpet", see Weber (2017).

If we assume again for simplicity that interest rates over a one-year horizon are zero, setting SCR := $\rho(X)$, we obtain the following solvency condition:

$$E_1 \in \mathcal{A}_\rho \quad \Leftrightarrow \quad \rho(E_1) \leq 0 \quad \Leftrightarrow \quad \text{SCR} \leq E_0.$$

The Swiss Solvency Test chooses AV@R as the basis for the definition of solvency.

Let us now return to the original question regarding the impact of the tax system on solvency capital. In eq. (1) we computed the tax savings that would accrue if a deduction of VAT paid on inputs was permitted. We assume that these savings of $\bar{L} - L = \tau_{VAT}\alpha\pi$ are retained by the insurance company. While initial capital E_0 remains unchanged, capital E_1 at the solvency time horizon is increased by this amount. This leads to a reduction of the SCR.

Lemma 7.3. *We denote by SCR the solvency capital requirement in the original tax system with premium tax. Assume that the tax system is modified such that a deduction of VAT paid on inputs is permitted. In this case, the solvency capital requirement is reduced to*

$$SCR - \tau_{VAT}\alpha\pi.$$

Proof. Adjusted quantities are labeled with a tilde. The random economic capital at time $t = 1$ becomes $\widetilde{E}_1 = E_1 + \tau_{VAT}\alpha\pi$. We compute

$$\widetilde{\text{SCR}} = \rho(X + \tau_{VAT}\alpha\pi) = \rho(X) - \tau_{VAT}\alpha\pi = \text{SCR} - \tau_{VAT}\alpha\pi.$$

□

Considering the situation in Example 2.1, the reduction of the solvency capital by $\tau_{VAT}\alpha\pi$ would amount to 0.99% of gross premium income. This is due to decreased government revenues. The company could increase the dividend payments to its shareholders by this amount. If this is done, the NAV at time 1 will return to its original level E_1. The solvency situation of the insurance company, i.e. $E_1 \in \mathcal{A}_\rho$, will then be the same as before. Conversely, if the NAV of an insurance firm at time 0, i.e. E_0, is close to the SCR in a tax system with VAT, the firm would need a capital injection of 0.99% of gross premium income from its shareholders to satisfy the same solvency capital constraint in a tax system with premium tax.

8 Conclusion

We analyzed the impact of premium tax on total tax revenues, insurance demand, the competitiveness of corporate buyers of insurance, the ruin probability of insurance firms and their solvency capital requirement. We find that the competitiveness of corporate buyers of insurance, the ruin probability of insurance firms and their solvency capital are hardly affected. In contrast, the tax system (i.e. premium tax vs. VAT) has a significant influence on the cost of insurance, insurance demand, government revenues and the profitability of insurance firms. The increased cost of insurance in tax systems with premium tax in contrast to VAT might promote alternative risk transfer mechanisms such as off-shore captive insurance, derivatives, or preventative measures that are not subject to premium tax. These instruments might provide more cost-efficient solutions to the risk management needs of corporations. On the one hand, some tax-efficient products might offer new business opportunities for insurance firms. On the other hand, alternative risk transfer mechanisms might also cannibalize their traditional business. The design of such instruments and their implications for corporate risk management, insurance companies and government revenues are interesting topics for further research.

A Proof of Theorem 4.4

Proof.

(i) CARA-utility:
We first consider the case $X \sim \text{Ber}(\hat{x}, p)$. We compute

$$\mathbb{E}[u_1^\kappa(X_\nu)] \;=\; 1 - e^{-\kappa(w-\nu\pi)} \left(e^{\kappa(1-\nu)\hat{x}} p + 1 - p\right).$$

This implies

$$\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)] \;=\; \kappa e^{\kappa(\pi\nu - \hat{x}\nu + \hat{x} - w)} \left(\pi(p-1)e^{\kappa\hat{x}\nu - \kappa\hat{x}} + p(\hat{x} - \pi)\right).$$

At the boundary $\nu = 0$ we obtain

$$\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)]_{|\nu=0} \;=\; \kappa e^{\kappa(\hat{x}-w)} \left(\pi(p-1)e^{-\kappa\hat{x}} + p(\hat{x} - \pi)\right).$$

Thus, $\nu(\pi) = 0$ is the optimal solution, if and only if

$$\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)]_{|\nu=0} \;\le\; 0 \quad\iff\quad \pi \;\ge\; \frac{p\hat{x}e^{\kappa\hat{x}}}{1 - p + pe^{\kappa\hat{x}}}.$$

At the boundary $\nu = 1$ we obtain $\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)]_{|\nu=1} = \kappa e^{\kappa(\pi-w)}(p\hat{x} - \pi)$. Thus, the optimal solution is $\nu(\pi) = 1$, iff $\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)]_{|\nu=1} \ge 0$, i.e. $\pi \le p\hat{x}$. In all other cases, we need to solve $\frac{\partial}{\partial\nu}\mathbb{E}[u_1^\kappa(X_\nu)] = 0$, leading to the stated solution. The first order conditions are sufficient due to the strict concavity.
Second, we derive the optimal contract for $X \sim \Gamma(\xi, \mu)$. In this case,

$$\begin{aligned}
\mathbb{E}\left[u_1^\kappa(X_\nu)\right] \;&=\; 1 - e^{-\kappa(w-\nu\pi)} \left(\frac{\mu}{\mu - \kappa(1-\nu)}\right)^{\xi}, \\
\frac{\partial}{\partial\nu}\mathbb{E}\left[u_1^\kappa(X_\nu)\right] \;&=\; e^{-\kappa(w-\nu\pi)} \left(\frac{\mu}{\mu - \kappa(1-\nu)}\right)^{\xi+1} \left(-\frac{\kappa}{\mu}\right)(\pi(\mu - \kappa(1-\nu)) - \xi).
\end{aligned}$$

The solution can now be derived by analogous arguments as before.

(ii) HARA-utility: Using the same steps as above, the solution is computed, observing

$$\begin{aligned}
\mathbb{E}\left[u_2^\lambda(X_\nu)\right] \;&=\; \frac{1}{\lambda}\cdot\left(((1-\nu)(w-\hat{x}) + \nu(w-\pi))^\lambda \cdot p + ((1-\nu)w + \nu(w-\pi))^\lambda \cdot (1-p)\right), \\
\frac{\partial}{\partial\nu}\mathbb{E}\left[u_2^\lambda(X_\nu)\right] \;&=\; (w - \nu\pi + \hat{x}(\nu - 1))^{\lambda-1} p(\hat{x} - \pi) + (-\pi)(1-p)(w - \nu\pi)^{\lambda-1}.
\end{aligned}$$

(iii) Logarithmic utility: Again, the solution is derived by analogous arguments, noting

$$\begin{aligned}
\mathbb{E}\left[u_2^0(X_\nu)\right] \;&=\; \log((1-\nu)(w-\hat{x}) + \nu(w-\pi)) \cdot p + \log((1-\nu)w + \nu(w-\pi)) \cdot (1-p), \\
\frac{\partial}{\partial\nu}\mathbb{E}\left[u_2^0(X_\nu)\right] \;&=\; \frac{1}{w - \nu\pi + \hat{x}\nu - \hat{x}} p(\hat{x} - \pi) + (-\pi)(1-p)\frac{1}{w - \nu\pi}.
\end{aligned}$$

□

B Computations of Section 2

Rate of Input

	2015	2014	2013	2012	2011
Gross premiums earned π	75,008,740	71,216,091	69,298,052	66,922,556	63,514,681
Capital income	7,431,575	7,246,143	7,207,242	7,451,052	6,988,566
Investment expenses	1,269,941	923,197	1,010,400	1,098,403	1,740,899
Total losses	56,243,800	52,078,719	55,722,781	50,255,508	48,929,622
Acquisition costs and administrative expenses	18,921,252	18,083,843	17,594,251	17,113,492	16,486,877
Taxes	1,462,200	1,479,200	963,100	1,483,700	1,147,200
Input L	4,543,122	5,897,275	1,214,762	4,422,505	2,198,649

Table 6: Computation of input costs in thousands of euros (TEUR).
These values are given in Table 540 of the corresponding annual report of BaFin (2011-2015). (Negative) Taxes are disclosed in Table 79 of the same reports.

	2015	2014	2013	2012	2011	Mean
Gross premiums earned	100.00	100.00	100.00	100.00	100.00	
Capital income	9.91	10.17	10.40	11.13	11.00	
Investment expenses	1.69	1.30	1.46	1.64	2.74	
Total losses	75.00	73.10	80.40	75.10	77.00	
Acquisition costs and administrative expenses	25.20	25.40	25.40	25.60	26.00	
Taxes	1.95	2.08	1.39	2.22	1.81	
Rate of Input α	6.07	8.30	1.75	6.58	3.46	5.23

Table 7: Computation of rate of input α as ratio of earned gross premium π.
Total losses as well as acquisition costs and administrative expenses can be adopted from Table 540 in the corresponding annual report of BaFin (2011-2015). Other quantities are calculated using Table 6.

Value added

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	18,921,252	18,083,843	17,594,251	17,113,492	16,486,877
Profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Gross technical result	4,859,078	5,076,151	-165,970	3,442,647	1,812,497
Net technical result	2,931,694	2,908,918	288,506	1,573,496	377,868
Value added $\widetilde{W}$	23,692,336	23,523,476	19,094,875	22,313,643	19,539,506

Table 8: Computation of value added in TEUR.
Profits before taxes and (negative) changes in equilization provisions are given in Table 79 of the corresponding annual report of BaFin (2011-2015). Gross and net technical results can be found in Table 540 of the same reports.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	25.20	25.40	25.40	25.60	26.00	
Profits before taxes	3.40	3.63	3.08	3.69	3.13	
Changes in equilization provisions	0.39	0.96	-0.26	1.28	-0.58	
Gross technical result	6.50	7.10	-0.20	5.10	2.90	
Net technical result	3.91	4.08	0.42	2.35	0.59	
Value added $\frac{\widetilde{W}}{\pi}$	31.58	33.01	27.60	33.33	30.85	31.28

Table 9: Computation of value added as ratio of earned gross premium π.
Acquisition costs and administrative expenses as well as gross technical result can be adopted from Table 540 in the corresponding annual report of BaFin (2011-2015). Other quantities are calculated using Table 8 and π in Table 6.

Value added for different lines of insurance
Accident:

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned accident (direct business) $\underline{\pi}^*$	6,388,854	6,440,961	6,416,895	6,500,627	6,383,714
Net premium earned accident (direct business) $\hat{\underline{\pi}}^*$	5,487,886	5,545,237	5,707,299	5,638,822	5,587,643
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result accident	24.70	16.60	17.10	17.20	17.30

Table 10: Needed data for computation of value added accident.
Total gross premium earned, gross premium earned accident, net premium earned accident and net technical result accident are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\hat{\underline{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	1,993,322	2,003,139	2,002,071	2,054,198	2,055,556
Profits before taxes	234,429	251,996	212,367	258,821	213,833
Changes in equilization provisions	27,175	66,653	-17,967	89,854	-39,684
Gross technical result	1,252,215	1,210,901	1,270,545	1,280,624	1,238,441
Net technical result	1,355,508	920,509	975,948	969,877	966,662
Value added accident	2,151,634	2,612,179	2,491,068	2,713,619	2,501,483

Table 11: Computation of value added accident in TEUR.
All quantities except net technical result are computed using Table 12 and $\underline{\pi}^*$ given in Table 10. Net technical result is calculated by multiplying the corresponding quantities in Table 10.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	31.20	31.10	31.20	31.60	32.20	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	19.60	18.80	19.80	19.70	19.40	
Net technical result	21.22	14.29	15.21	14.92	15.14	
Value added accident	33.68	40.56	38.82	41.74	39.19	38.80

Table 12: Computation of value added accident as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 10. Net technical results are based on the absolute values in Table 11 and $\underline{\pi}^*$ given in Table 10.

Public Liability (publicL):

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned publicL (direct business) $\underline{\pi}^*$	9,246,435	8,837,457	8,360,776	8,023,858	7,706,079
Net premium earned publicL (direct business) $\underline{\hat{\pi}}^*$	6,714,540	6,536,798	6,670,263	6,437,008	5,979,624
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result publicL	7.50	9.00	7.20	9.80	9.80

Table 13: Needed data for computation of value added public liability.
Total gross premium earned, gross premium earned public liability, net premium earned public liability and net technical result public liability are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\underline{\hat{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	2,810,916	2,695,424	2,575,119	2,527,515	2,450,533
Profits before taxes	339,283	345,756	276,700	319,468	258,127
Changes in equilization provisions	39,330	91,453	-23,410	110,908	-47,904
Gross technical result	674,990	821,884	627,058	866,577	608,780
Net technical result	503,591	588,312	480,259	630,827	586,003
Value added public liability	3,360,929	3,366,205	2,975,208	3,193,641	2,683,533

Table 14: Computation of value added public liability in TEUR.
All quantities except net technical result are computed using Table 15 and $\underline{\pi}^*$ given in Table 13. Net technical result is calculated by multiplying the corresponding quantities in Table 13.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	30.40	30.50	30.80	31.50	31.80	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	7.30	9.30	7.50	10.80	7.90	
Net technical result	5.45	6.66	5.74	7.86	7.60	
Value added public liability	36.35	38.09	35.59	39.80	34.82	36.93

Table 15: Computation of value added public liability as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 13. Net technical results are based on the absolute values in Table 14 and $\underline{\pi}^*$ given in Table 13.

Car Total:

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned car (direct business) $\underline{\pi}^*$	24,601,179	23,637,844	22,503,977	21,234,566	20,113,638
Net premium earned car (direct business) $\underline{\hat{\pi}}^*$	19,146,675	18,312,796	18,352,347	17,317,716	16,402,766
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result car total	2.00	3.70	-2.80	-3.30	-8.10

Table 16: Needed data for computation of value added car total.
Total gross premium earned, gross premium earned car total, net premium earned car total and net technical result car total are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\underline{\hat{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	4,206,802	4,089,347	3,960,700	3,822,222	3,640,568
Profits before taxes	902,702	924,806	744,768	845,449	673,738
Changes in equilization provisions	104,642	244,611	-63,011	293,510	-125,035
Gross technical result	615,029	827,325	-1,012,679	-509,630	-1,528,636
Net technical result	382,934	677,573	-513,866	-571,485	-1,328,624
Value added car total	5,446,241	5,408,515	4,143,643	5,023,036	3,989,259

Table 17: Computation of value added car total in TEUR.
All quantities except net technical result are computed using Table 18 and $\underline{\pi}^*$ given in Table 16. Net technical result is calculated by multiplying the corresponding quantities in Table 16.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	17.10	17.30	17.60	18.00	18.10	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	2.50	3.50	-4.50	-2.40	-7.60	
Net technical result	1.56	2.87	-2.28	-2.69	-6.61	
Value added car total	22.14	22.88	18.41	23.65	19.83	21.38

Table 18: Computation of value added car total as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 16. Net technical results are based on the absolute values in Table 17 and $\underline{\pi}^*$ given in Table 16.

Defense:

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned defense (direct business) $\underline{\pi}^*$	3,949,994	3,824,287	3,756,450	3,695,395	3,401,014
Net premium earned defense (direct business) $\hat{\underline{\pi}}^*$	3,440,597	3,317,429	3,367,084	3,306,620	3,048,240
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result defense	0.50	-0.40	0.50	3.50	3.40

Table 19: Needed data for computation of value added defense.
Total gross premium earned, gross premium earned defense, net premium earned defense and net technical result defense are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\hat{\underline{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	1,319,298	1,269,663	1,228,359	1,249,044	1,091,725
Profits before taxes	144,939	149,621	124,320	147,131	113,922
Changes in equilization provisions	16,801	39,575	-10,518	51,079	-21,142
Gross technical result	31,600	-22,946	7,513	136,730	112,233
Net technical result	17,203	-13,270	16,835	115,732	103,640
Value added defense	1,495,435	1,449,183	1,332,838	1,468,251	1,193,099

Table 20: Computation of value added defense in TEUR.
All quantities except net technical result are computed using Table 21 and $\underline{\pi}^*$ given in Table 19. Net technical result is calculated by multiplying the corresponding quantities in Table 19.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	33.40	33.20	32.70	33.80	32.10	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	0.80	-0.60	0.20	3.70	3.30	
Net technical result	0.44	-0.35	0.45	3.13	3.05	
Value added defense	37.86	37.89	35.48	39.73	35.08	37.21

Table 21: Computation of value added defense as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 19. Net technical results are based on the absolute values in Table 20 and $\underline{\pi}^*$ given in Table 19.

Fire:

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned fire (direct business) $\underline{\pi}^*$	2,150,739	1,888,463	1,840,158	1,736,250	1,763,792
Net premium earned fire (direct business) $\underline{\hat{\pi}}^*$	1,131,264	1,091,608	1,050,167	1,048,086	1,064,826
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result fire	-13.30	-7.50	-4.20	-10.20	-7.50

Table 22: Needed data for computation of value added fire.
Total gross premium earned, gross premium earned fire, net premium earned fire and net technical result fire are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\underline{\hat{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	615,111	523,104	506,043	503,513	502,681
Profits before taxes	78,918	73,884	60,900	69,128	59,081
Changes in equilization provisions	9,148	19,542	-5,152	23,999	-10,964
Gross technical result	-204,320	-18,885	58,885	-83,340	-15,874
Net technical result	-150,458	-81,871	-44,107	-106,905	-79,862
Value added fire	649,315	679,517	664,783	620,205	614,785

Table 23: Computation of value added fire in TEUR.
All quantities except net technical result are computed using Table 24 and $\underline{\pi}^*$ given in Table 22. Net technical result is calculated by multiplying the corresponding quantities in Table 22.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	28.60	27.70	27.50	29.00	28.50	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	-9.50	-1.00	3.20	-4.80	-0.90	
Net technical result	-7.00	-4.34	-2.40	-6.16	-4.53	
Value added fire	30.19	35.98	36.13	35.72	34.86	34.58

Table 24: Computation of value added fire as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 22. Net technical results are based on the absolute values in Table 23 and $\underline{\pi}^*$ given in Table 22.

Household:

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned househ. (direct business) $\underline{\pi}^*$	2,814,327	2,742,306	2,683,368	2,622,915	2,578,722
Net premium earned househ. (direct business) $\underline{\hat{\pi}}^*$	2,426,927	2,370,466	2,434,656	2,375,543	2,330,813
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result household	15.30	12.30	14.40	14.70	16.30

Table 25: Needed data for computation of value added household.
Total gross premium earned, gross premium earned household, net premium earned household and net technical result household are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\underline{\hat{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	982,200	959,807	936,495	925,889	902,553
Profits before taxes	103,267	107,290	88,806	104,431	86,378
Changes in equilization provisions	11,971	28,378	-7,513	36,255	-16,030
Gross technical result	484,064	394,892	402,505	445,896	477,064
Net technical result	371,320	291,567	350,590	349,205	379,923
Value added household	1,210,183	1,198,800	1,069,703	1,163265	1,070,042

Table 26: Computation of value added household in TEUR.
All quantities except net technical result are computed using Table 27 and $\underline{\pi}^*$ given in Table 25. Net technical result is calculated by multiplying the corresponding quantities in Table 25.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	34.90	35.00	34.90	35.30	35.00	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	17.20	14.40	15.00	17.00	18.50	
Net technical result	13.19	10.63	13.07	13.31	14.73	
Value added household	43.00	43.72	39.86	44.35	41.50	42.49

Table 27: Computation of value added household as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 25. Net technical results are based on the absolute values in Table 26 and $\underline{\pi}^*$ given in Table 25.

Residential Building (ResBui):

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned ResBui (direct business) $\underline{\pi}^*$	6,144,732	5,782,479	5,388,303	5,033,876	4,764,973
Net premium earned ResBui (direct business) $\hat{\underline{\pi}}^*$	4,702,787	4,425,298	4,329,537	4,064,797	3,834,273
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result residential building	-9.40	-8.30	-22.20	-12.50	-14.40

Table 28: Needed data for computation of value added residential building.
Total gross premium earned, gross premium earned residential building, net premium earned residential building and net technical result residential building are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\hat{\underline{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	1,720,525	1,624,877	1,530,278	1,424,587	1,362,782
Profits before taxes	225,471	226,233	178,326	200,423	159,610
Changes in equilization provisions	26,137	59,839	-15,087	69,580	-29,621
Gross technical result	-147,474	-138,779	-1,929,012	-241,626	-385,963
Net technical result	-442,062	-367,300	-961,157	-508,100	-552,135
Value added residential building	2,266,721	2,139,469	725,661	1,961,063	1,658,944

Table 29: Computation of value added residential building in TEUR.
All quantities except net technical result are computed using Table 30 and $\underline{\pi}^*$ given in Table 28. Net technical result is calculated by multiplying the corresponding quantities in Table 28.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	28.00	28.10	28.40	28.30	28.60	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	-2.40	-2.40	-35.80	-4.80	-8.10	
Net technical result	-7.19	-6.35	-17.84	-10.09	-11.59	
Value added residential building	36.89	37.00	13.47	38.96	34.82	32.23

Table 30: Computation of value added residential building as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 28. Net technical results are based on the absolute values in Table 29 and $\underline{\pi}^*$ given in Table 28.

Credit and Guarantee (CreGua):

	2015	2014	2013	2012	2011
Total gross premium earned (direct business) $\underline{\pi}$	69,448,394	66,146,203	64,535,515	62,102,602	59,310,517
Gross premium earned CreGua (direct business) $\underline{\pi}^*$	450,905	415,173	988,984	958,490	1,235,832
Net premium earned CreGua (direct business) $\hat{\underline{\pi}}^*$	429,550	403,996	585,225	560,561	845,929
Total profits before taxes	2,548,300	2,587,900	2,135,800	2,472,600	1,986,700
Total changes in equilization provisions	295,400	684,500	-180,700	858,400	-368,700
Net technical result credit and guarantee	32.40	29.10	20.20	23.00	29.20

Table 31: Needed data for computation of value added credit and guarantee.
Total gross premium earned, gross premium earned credit and guarantee, net premium earned credit and guarantee and net technical result credit and guarantee are given in Table 541 of the corresponding annual report of BaFin (2011-2015). As above, total profits before taxes and total (negative) changes in equilization provisions can be adopted from Table 79 in the same reports. Positions 1 to 5 are specified in TEUR, position 6 as ratio of $\hat{\underline{\pi}}^*$.

	2015	2014	2013	2012	2011
Acquisition costs and administrative expenses	132,115	119,155	273,949	272,211	359,627
Profits before taxes	16,545	16,243	32,730	38,162	41,396
Changes in equilization provisions	1,918	4,296	-2,769	13,249	-7,682
Gross technical result	124,450	126,213	206,698	150,483	411,532
Net technical result	139,174	117,563	118,215	128,929	247,011
Value added credit and guarantee	135,854	148,344	392,392	345,176	557,862

Table 32: Computation of value added credit and guarantee in TEUR.
All quantities except net technical result are computed using Table 33 and $\underline{\pi}^*$ given in Table 31. Net technical result is calculated by multiplying the corresponding quantities in Table 31.

	2015	2014	2013	2012	2011	Mean
Acquisition costs and administrative expenses	29.30	28.70	27.70	28.40	29.10	
Profits before taxes	3.67	3.91	3.31	3.98	3.35	
Changes in equilization provisions	0.43	1.03	-0.28	1.38	-0.62	
Gross technical result	27.60	30.40	20.90	15.70	33.30	
Net technical result	30.87	28.32	11.95	13.45	19.99	
Value added credit and guarantee	30.13	35.73	39.68	36.01	45.14	37.34

Table 33: Computation of value added credit and guarantee as ratio of earned gross premium $\underline{\pi}^*$.
Acquisition costs and administrative expenses as well as gross technical results are given in Table 541 of the corresponding annual report of BaFin (2011-2015). Profits before taxes and changes in equilization provisions are calculated by using total values in Table 31. Net technical results are based on the absolute values in Table 32 and $\underline{\pi}^*$ given in Table 31.

Note: When data were available from different BaFin-reports, we have always chosen the most current data source. Whenever possible, we used data from BaFin (2011-2015), Issue 2015, for the years 2013-2015, BaFin (2011-2015), Issue 2014, for the year 2012 and BaFin (2011-2015), Issue 2013, for the year 2011. This refers to Table 540 in the corresponding issues. For the year 2014 we had to rely on Table 80 instead of Table 79 in BaFin (2011-2015), Issue 2014.

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