Regime Switching Rough Heston Model

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4 Conclusion
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Motivations & Stylized Facts

(a) Observed volatility regimes

(b) Log return–realized vol

(c) BM sample paths

(d) Rough paths
Stochastic Volatility Modeling

- Fix the time horizon \([0, T]\) and assume the usual filtered probability space 
  \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 \leq t \leq T})\).
- The most common stochastic volatility model is the Heston [1993] (under \(\mathbb{Q}\)) defined as follows:
  \[
  \begin{align*}
  dS_t &= rS_t dt + S_t \sqrt{V_t} dB_t \\
  dV_t &= \kappa (\theta - V_t) dt + \sigma \sqrt{V_t} dW_t,
  \end{align*}
  \]
  (1)
  - with \(\langle dB_t, dW_t \rangle = \rho dt\).
  - In this formulation, leverage effects and mean-reverting property of volatility are both captured.
  \[2\kappa \theta \geq \sigma^2 \Rightarrow \mathbb{P} \left( \exists t \in [0, T] \text{ with } V_t = 0 \right) = 0. \text{ a.s.}\]
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Regimes In Volatility

- Derman [1999] empirically proved that the term structure of implied volatility has regimes.
- Maghrebi et al. [2014] statistically showed that a model should have at least two regimes under the pricing measure.
- Elliott et al. [2016] considered only $\theta(t)$ modulated by a Markov chain $(Z_t)$ with $n$-possible states:

$$\theta_t = \{\bar{\theta}^1, \bar{\theta}^2, \cdots, \bar{\theta}^n\},$$

- with transition rate matrix:

$$\Gamma = (\gamma_{i,j})_{i,j=1}^n$$

- where:

$$\text{for all } i \neq j \gamma_{i,j} \Delta t = \mathbb{P}(\theta_{t+\Delta t} = \bar{\theta}^j | \theta_t = \theta_i)$$

- Elliott and his coauthors proved that Markov Switching SV better fit implied vol while preserving analytical tractability.
And Volatility Is Rough

- Recently Gatheral et al. [2018] discovered a universal phenomenon that volatility is rough.
- These observations led to application of fBM processes. This goes back to the framework of Comte and Renault [1998].
- Consider Mandelbrot and Van Ness [1968] representation for the fBm:

\[
W_t^H = \frac{1}{\Gamma (H + \frac{1}{2})} \int_0^t \frac{1}{(t-s)^{\frac{1}{2}-H}} dW_s + \frac{1}{\Gamma (H + \frac{1}{2})} \int_{-\infty}^0 \left[ \frac{1}{(t-s)^{\frac{1}{2}-H}} - \frac{1}{(-s)^{\frac{1}{2}-H}} \right] dW_s, \quad H \in (0, 1].
\]

(2)
- For all \( \epsilon > 0 \), and \( H \) very small, the fractional kernel

\[
\frac{1}{(t-s)^{\frac{1}{2}-H}}
\]

is behind the \( H - \epsilon \) Hölder regularity of volatility [Euch and Rosenbaum, 2017].
Volatility Is Rough Indeed

- There is a literature that develops models that can cope with this feature observed in the data.
- There is a bulk of literature out there, e.g. Gatheral et al. [2018], Bayer et al. [2016], [Euch and Rosenbaum, 2016, 2017], Fukasawa [2017], Alfeus et al. [2017], and Callegaro et al. [2018]
- https://sites.google.com/site/roughvol/home/risks-1
- Key papers ↪ Euch and Rosenbaum [2016, 2017]
The Rough framework introduces roughness paths in the volatility process of Heston model - Roughening Heston.

Let \( X_t = \log S_t \).

The Rough Heston model (under \( \mathbb{Q} \)) is given by:

\[
\begin{align*}
    dX_t &= (r - V_t/2)dt + \sqrt{V_t}dB_t \\
    V_t &= V_0 + \frac{\kappa}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}(\theta - V_s)dt + \frac{\sigma}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\sqrt{V_s}dW_s
\end{align*}
\]

\[\alpha = \frac{1}{2} + H \in (0.5, 1).\]

Intricate issues

- (3) is neither Markovian nor a semi-martingale

\[\alpha = \frac{1}{2} + H \in (0.5, 1).\]
Euch and Rosenbaum [2016] Framework II

- El Euch and Rosenbaum construct the solution of this process by passing this through a scaling limit of a sequences of Hawkes processes.
- A useful description of its law is found by going through fractional calculus.
- They derived explicit form for the characteristic function in exponential affine expression.
- This was made possible mainly due to the scaling limit of the Hawkes processes
- This model is proven to capture the both the behaviour of Historical and implied volatility.
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Our approach

• To extend the arguments from Euch and Rosenbaum [2016] as well as from Elliott et al. [2016]

• To build unified and consistent framework that capture two important stylized features of volatility:
  • The rough behaviour in its local behaviour
  • The regime switching property consistent with more long term economic consideration

• We derive an analytic representation of the Laplace-functional of the asset price.

• We benchmark these semi-analytic prices against two types of Monte-Carlo-simulations:
  1. One is a full Monte-Carlo simulation, in which the three dimensional stochastic processes \((B, W, \theta)\).
  2. a novel method in this context, is the partial Monte-Carlo-Simulation. Here we simulate the path of \(\theta_s(\omega), s \in [0, T]\) and then solve the corresponding rough Riccati equation.
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Fixed function $s \rightarrow \theta_s$

- The Rough volatility framework in Equation (3) now becomes

$$
\begin{align*}
\frac{dX_t}{dt} &= (r - V_t/2)dt + \sqrt{V_t}dB_t \\
V_t &= V_0 + \frac{\kappa}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1}(\theta(s) - V_s)dt + \frac{\sigma}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1}\sqrt{V_s}dW_s.
\end{align*}
$$

If

$$
\forall \epsilon > 0 \exists C_\epsilon > 0; \forall u \in (0, T]; |\theta(u)| \leq C_\epsilon u^{-\frac{1}{2}-\epsilon},
$$

then there is a weak non-negative solution exhibiting $H - \epsilon$ Hölder regularity.
Regime switching $\theta_s$

- To incorporate regime switching into the mean reversion level $\theta$ as in Elliott et al. [2016], we define a finite-state time homogeneous Markov process

$$\theta_s(\omega) = \sum_{i=1}^{k} \vartheta_i Z_s^{(i)}(\omega) = \langle \vartheta, Z_s \rangle,$$

with generator matrix $Q$ and where $Z$ is a Markov chain, independent from $(S, V)$ with state space the set of unit vectors in $\mathbb{R}^k$, i.e. $Z_s \in \{e_i = (0, \ldots, 1, 0, \ldots)^T, i = 1, \ldots, k\}$ and $\vartheta$ is the vector of $k$-different mean reversion levels.

- The dynamics of $Z_t$ is given by

$$dZ_t = Q' Z_t dt + dM_t$$ (5)
Theorem

Let $\varphi_{X_T}(u; V_0, Z_0, X_0)$ be the characteristic function of $X_T$. The following expression holds:

$$\varphi_{X_T}(u) = E[e^{iuX_T}] = E\left[\exp\left(\kappa \int_0^T h(u, T-s)\langle \vartheta, Z_s \rangle ds\right)\right] e^{\int_0^T h(u, s) \frac{V_0 s^{-\alpha}}{\Gamma(1-\alpha)} ds}. \quad (6)$$

where $h$ is the unique solution of the following fractional Riccati equation:

$$D^\alpha h = \frac{1}{2}(-u^2 - iu) + (iu\rho\sigma - \kappa)h(u, s) + \frac{\sigma^2}{2} h^2(u, s), s < t, u \in \mathbb{C}, \quad (7)$$

$$I^{1-\alpha} h(u, 0) = 0.$$

Here the Riemann-Liouville fractional differentiation and integral are defined by

$$D^\alpha h(u, s) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} h(u, s) ds, \quad I^\alpha h(u, s) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} h(u, s) ds.$$
Computation of the Characteristic Function I

• The computational complexity:

\[
E[e^{iuXT}] = E\left[\exp \left(\kappa \int_0^T h(u, T-s)\langle \vartheta, Z_s \rangle ds\right)\right] \left(\begin{array}{c}
E^\left(\begin{array}{c}
\exp \left(\kappa \int_0^T h(u, T-s)\langle \vartheta, Z_s \rangle ds\right)\right)
\end{array}\right)^{(b)}
\right)
\]

\[e^{\int_0^T h(u, T-s)\frac{V_0 s^{-\alpha}}{\Gamma(1-\alpha)} ds}. \quad (8)\]

• The fractional differential Ricatti equation is solved using a predictor-corrector approach (Adams schemes)
• A new technology based on hybrid schemes is introduced by Callegaro et al. [2018]
• Diethelm et al. [2004] presents a numerical algorithm for evaluating the fractional integral (b).
The computation of the expectation \((a)\) in (8):

- Fix \(T\). Let

\[
g_t = \exp \left( \kappa \int_0^t h(u, T-s) \langle \vartheta, Z_s \rangle \, ds \right), \quad 0 < t \leq T
\]  

\( (9) \)

\[
G_t := g_t Z_t.
\]  

\( (10) \)

- Then

\[
dG_t = g_t dZ_t + Z_t dg_t
\]  

\( (11) \)

\[
= g_t (Q' Z_t dt + dM_t) + Z_t g_t \kappa h(u, T-t) \langle \vartheta, Z_t \rangle dt
\]

\[
= (Q' + \kappa h(u, T-t) \Theta) g_t Z_t dt + g_t dM_t
\]

- Consider

\[
\hat{G}_t = \mathbb{E}[G_t | Z_0 = Z] = \Phi(s,t) Z.
\]
Matrix ODE

• Along the lines of Elliott et al. [2016], we showed that since

$$\mathbb{E}[G_t] = \langle \hat{G}_t, 1 \rangle$$

• Then

$$E \left[ \exp \left( \kappa \int_0^T h(u, T - s) \langle \vartheta, Z_s \rangle \, ds \right) \right] = \langle \Phi(0, T)Z_0, 1 \rangle,$$

• where $\Phi$ is a solution of the matrix ODE given by

$$\frac{d\Phi(s, t)}{dt} = \left( Q' + \kappa h(u, T - t) \Theta \right) \Phi(s, t), \ s \leq t, \ \text{with} \ \Phi(s, s) = I, \quad (12)$$

• where $\Theta = \text{diag}[\vartheta]$. 
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Semi-Analytic Pricing Formula

- To price options, we use the well-known Fourier-inversion formula of Gil-Pelaez [1951] which leads to a semi-analytic closed-form solution given by:

\[ C_0 = e^{-rT} \mathbb{E} [(e^X - K)^+] = \mathbb{E}[e^X] \Pi_1 - e^{-rT} K \Pi_2, \tag{13} \]

where the probability quantities \( \Pi_1 \) and \( \Pi_2 \) are given by:

\[
\Pi_1 = \frac{\mathbb{E}[e^X \mathbb{I}_{\{e^X > K\}}]}{\mathbb{E}[e^X]} = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{e^{-iu \log(K)} \varphi_X(u - i)}{iu \varphi_X(-i)} \right] dz \tag{14}
\]

\[
\Pi_2 = \mathbb{P}\{e^X > K\} = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{e^{-iu \log(K)} \varphi_X(u)}{iu} \right] dz.
\]
Monte Carlo Approaches

- Two Monte Carlo simulations have been implemented:
  1. Full Monte Carlo
     - We first simulate the 3-dimensional process $(B, W, \theta)$.
     - The option pay-out can be obtained (in the risk neutral world) in each simulation.
  2. Partial Monte Carlo
     - This is a novel method in this context
     - We simulate the paths of $\theta_s$ and then evaluate for each realization $\theta_s(\omega)$, the formula (8), i.e., for $l = 1, 2, \ldots, N$

$$E[e^{iuX_t}](\omega_l) = \exp \left( \int_0^t h(u, T - s) \left( \kappa \theta_s(\omega_l) + \frac{V_0 s^{-\alpha}}{\Gamma(1 - \alpha)} \right) ds \right), \quad (15)$$

- Then

$$E[e^{iuX_t}] \sim \frac{1}{N} \sum_{l=1}^{N} E[e^{iuX_t}](\omega_l) \quad (16)$$
# The Pricing Parameters

Table: Adopted from Elliott et al. [2016]

<table>
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<th>Parameters</th>
<th>value</th>
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<tr>
<td>( r )</td>
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<tr>
<td>( \sigma )</td>
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<td>( \rho )</td>
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<td>( \kappa )</td>
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</tr>
<tr>
<td>( \theta_0 = [\theta^1 \ \theta^2] )</td>
<td>[0.025 \ 0.075]</td>
</tr>
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</table>
| \( Q_E \)   | \[
-1 \ 1 \\
0.5 \ -0.5
\] |
| No. of Simulations | 1 000 000   |
| Time Steps       | 250         |
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Call prices, $v_0 = 0.02 < \theta^1 < \theta^2$, $H = 0.5$

(a) Starting in a low state: $\theta_0 = \theta^1$

<table>
<thead>
<tr>
<th>K/T</th>
<th>Monte Carlo</th>
<th>Semi-Analytic</th>
<th>Monte Carlo</th>
<th>Semi-Analytic</th>
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<td>Price</td>
<td>std Error</td>
<td>Price</td>
<td>std Error</td>
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<tr>
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(b) Starting in a high state: $\theta_0 = \theta^2$

<table>
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</tbody>
</table>
Call prices, $v_0 = 0.02 < \theta^1 < \theta^2$, $H = 0.1$

(a) Starting in a low state: $\theta_0 = \theta^1$

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<td>Full std Error Partial</td>
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Call prices, $v_0 = 0.02 < \theta^1 < \theta^2$, $H = 0.1, Q > Q_E$

(a) Starting in a low state: $\theta_0 = \theta^1$

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(b) Starting in a high state: $\theta_0 = \theta^2$

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Call prices - Heston model modifications, $T = 1$

(a) Starting in a low state: $\theta_0 = \theta^1$

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(b) Starting in a high state: $\theta_0 = \theta^2$

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Hurst Parameter Price Impact

(a) 6 months maturity

(b) 1 year maturity

(c) 1.85 years maturity

(d) 2.5 years maturity
Implied Volatility Surface

Implied Volatility - Regime Switching Rough Heston Model

- $H=0.1$, low regime
- $H=0.3$, high regime
- $H=0.5$, high regime

Ludger Overbeck, University of Giessen, Germany
2019 Workshop on Insurance and Financial Mathematics, Talanx, Hannover
## Computational Speed

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<th>Pricing Method</th>
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*Time measured in seconds.*
Concluding Remarks

• We studied the regime switching rough Heston models.
• The main goal is to construct a unified framework that captures the stylized facts of volatility.
• We developed a pricing engine and fully implemented this analytic approach.
• We benchmark these semi-analytic prices against two types of Monte-Carlo-simulations.
• Talk is based on the paper with Mesias Alfeus and Erik Schloegl: ”Regime switching rough Heston model” in Journal of Futures Markets Volume 39, Issue 5. 2019
Thank you!
References


