Regime Switching Rough Heston Model

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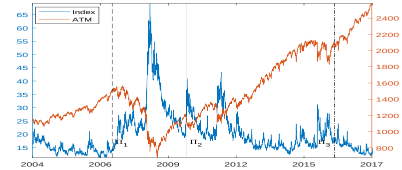
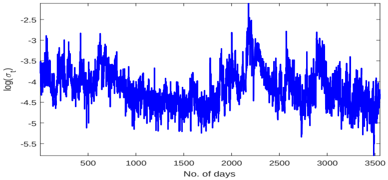
Outline

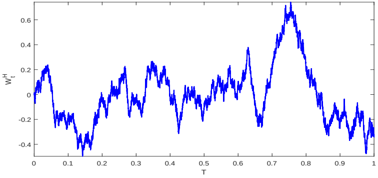
Motivations & Stylized Facts



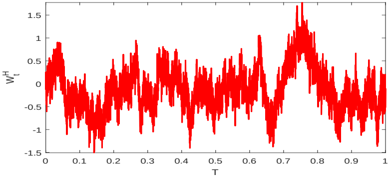
(a) Observed volatility regimes



(b) Log return–realized vol



(c) BM sample paths



(d) Rough paths

Stochastic Volatility Modeling

- Fix the time horizon $[0, T]$ and assume the usual filtered probability space

$$(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 \leq t \leq T}) .$$

- The most common stochastic volatility model is the Heston [1993] (under $\mathbb{Q}$) defined as follows:

$$\begin{aligned} dS_t &= rS_t dt + S_t \sqrt{V_t} dB_t \\ dV_t &= \kappa(\theta - V_t) dt + \sigma \sqrt{V_t} dW_t, \end{aligned} \tag{1}$$

  - with $\langle dB_t, dW_t \rangle = \rho dt$.
  - In this formulation, leverage effects and mean-reverting property of volatility are both captured.

$$2\kappa\theta \geq \sigma^2 \Rightarrow \mathbb{P}\left(\exists t \in [0, T] \text{ with } V_t = 0\right) = 0. \text{ a.s.}$$

Regimes In Volatility

- Derman [1999] empirically proved that the term structure of implied volatility has regimes
- Maghrebi et al. [2014] statistically showed that a model should have at least two regimes under the pricing measure
- Elliott et al. [2016] considered only $\theta(t)$ modulated by a Markov chain (Z_t) with n-possible states

$$\theta_t = \{\bar{\theta}^1, \bar{\theta}^2, \cdots, \bar{\theta}^n\},$$

- with transition rate matrix

$$\Gamma = (\gamma_{i,j})_{i,j=1}^n$$

- where

$$\text{for all } i \neq j \ \gamma_{ij}\Delta t = \mathbb{P}\left(\theta_{t+\Delta t} = \bar{\theta}^j | \theta_t = \theta_i\right)$$

- Elliott and his coauthors proved that Markov Switching SV better fit implied vol while preserving analytical tractability.

... And Volatility Is Rough

- Recently Gatheral et al. [2018] discovered a universal phenomenon that volatility is rough
- These observations led to application of fBM processes. This goes back to the framework of Comte and Renault [1998].
- Consider Mandelbrot and Van Ness [1968] representation for the fBm:

$$W_t^H = \frac{1}{\Gamma\left(H+\frac{1}{2}\right)}\int_0^t \frac{1}{(t-s)^{\frac{1}{2}-H}}dW_s + \frac{1}{\Gamma\left(H+\frac{1}{2}\right)}\int_{-\infty}^0 \left[\frac{1}{(t-s)^{\frac{1}{2}-H}} - \frac{1}{(-s)^{\frac{1}{2}-H}}\right]dW_s,\ H \in (0,1]. \tag{2}$$

- For all $\epsilon > 0$, and H very small, the fractional kernel

$$\frac{1}{(t-s)^{\frac{1}{2}-H}}$$

is behind the $H-\epsilon$ Hölder regularity of volatility [Euch and Rosenbaum, 2017]

Volatility Is Rough Indeed

- There is a literature that develops models that can cope with this feature observed in the data.
- There is a bulk of literature out there, e.g. Gatheral et al. [2018], Bayer et al. [2016], [Euch and Rosenbaum, 2016, 2017], Fukasawa [2017], Alfeus et al. [2017], and Callegaro et al. [2018]
- https://sites.google.com/site/roughvol/home/risks-1
- Key papers $\hookrightarrow$ **Euch and Rosenbaum [2016, 2017]**

Euch and Rosenbaum [2016] Framework I

- The Rough framework introduces roughness paths in the volatility process of Heston model - Roughening Heston.
- Let $X_t = \log S_t$.
- The Rough Heston model (under $\mathbb{Q}$) is given by:

$$
\begin{aligned}
dX_t &= (r - V_t/2)dt + \sqrt{V_t}dB_t \\
V_t &= V_0 + \frac{\kappa}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1}(\theta - V_s)dt + \frac{\sigma}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1}\sqrt{V_s}dW_s
\end{aligned} \tag{3}
$$

$\alpha = \frac{1}{2} + H \in (0.5, 1)$.

- Intricate issues
  - (3) is neither Markovian nor a semi-martingale

Euch and Rosenbaum [2016] Framework II

- El Euch and Rosenbaum construct the solution of this process by passing this through a scaling limit of a sequences of Hawkes processes.
- A useful description of its law is found by going through fractional calculus.
- They derived explicit form for the characteristic function in exponential affine expression.
- This was made possible mainly due to the scaling limit of the Hawkes processes
- This model is proven to capture the both the behaviour of Historical and implied volatility.

Our approach

- To extend the arguments from **Euch and Rosenbaum [2016]** as well as from **Elliott et al. [2016]**
- To build unified and consistent framework that capture two important stylized features of volatility:
  - The rough behaviour in its local behaviour
  - The regime switching property consistent with more long term economic consideration
- We derive an analytic representation of the Laplace-functional of the asset price.
- We benchmark these semi-analytic prices against two types of Monte-Carlo-simulations:
  1. One is a full Monte-Carlo simulation, in which the three dimensional stochastic processes (B, W, θ).
  2. a novel method in this context, is the partial Monte-Carlo-Simulation. Here we simulate the path of $\theta_s(\omega), s \in [0, T]$ and then solve the corresponding rough Riccati equation.

Fixed function $s \to \theta_s$

- The Rough volatility framework in Equation (3) now becomes

$$\begin{aligned} dX_t &= (r - V_t/2)dt + \sqrt{V_t}dB_t \\ V_t &= V_0 + \frac{\kappa}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1}(\theta(s) - V_s)dt + \frac{\sigma}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1}\sqrt{V_s}dW_s. \end{aligned} \tag{4}$$

If

$$\forall \epsilon > 0 \; \exists C_\epsilon > 0; \; \forall u \in (0,T]; \;\; |\theta(u)| \leq C_\epsilon u^{-\frac{1}{2}-\epsilon},$$

then there is a weak non-negative solution exhibiting $H - \epsilon$ Hölder regularity.

Regime switching θ_s

- To incorporate regime switching into the mean reversion level θ as in Elliott et al. [2016], we define a finite-state time homogeneous Markov process

$$\theta_s(\omega) = \sum_{i=1}^{k} \vartheta_i Z_s^{(i)}(\omega) = \langle \boldsymbol{\vartheta}, Z_s \rangle,$$

  with generator matrix Q and where Z is a Markov chain, independent from (S, V) with state space the set of unit vectors in $\mathbb{R}^k$, i.e.
  $Z_s \in \{e_i = (0, .., 1, 0..)^T, i = 1, .., k\}$ and ϑ is the vector of k-different mean reversion levels.
- The dynamics of Z_t is given by

$$dZ_t = Q' Z_t dt + dM_t \tag{5}$$

Laplace Representation

Theorem

Let $\varphi_{X_T}(u; V_0, Z_0, X_0)$ be the characteristic function of X_T. The following expression holds:

$$\varphi_{X_T}(u) = E[e^{\mathrm{i}uX_T}] \quad = \quad E\left[\exp\left(\kappa \int_0^T h(u, T-s)\langle \boldsymbol{\vartheta}, Z_s\rangle ds\right)\right] e^{\int_0^T h(u,s)\frac{V_0 s^{-\alpha}}{\Gamma(1-\alpha)} ds}. \tag{6}$$

where h is the unique solution of the following fractional Riccati equation:

$$\begin{aligned} D^\alpha h \quad &= \quad \frac{1}{2}(-u^2 - \mathrm{i}u) + (\mathrm{i}u\rho\sigma - \kappa)h(u,s) + \frac{\sigma^2}{2}h^2(u,s), s < t, u \in \mathbb{C}, \\ I^{1-\alpha}h(u,0) \quad &= \quad 0. \end{aligned} \tag{7}$$

Here the Riemann-Liouville fractional differentiation and integral are defined by

$$D^\alpha h(u,s) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} h(u,s) ds, \quad I^\alpha h(u,s) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} h(u,s) ds$$

Computation of the Characteristic Function I

- The computational complexity:

$$E[e^{\mathrm{i}uX_T}] = \underbrace{E\left[\exp\left(\kappa\int_0^T h(u,T-s)\langle\boldsymbol{\vartheta}, Z_s\rangle ds\right)\right]}_{(a)} \overbrace{e^{\int_0^T h(u,T-s)\frac{V_0 s^{-\alpha}}{\Gamma(1-\alpha)}ds}}^{(b)}. \tag{8}$$

  - The fractional differential Ricatti equation is solved using a predictor-corrector approach (Adams schemes)
  - A new technology based on hybrid schemes is introduced by Callegaro et al. [2018]
  - Diethelm et al. [2004] presents a numerical algorithm for evaluating the fractional integral (b).

Computation of the Characteristic Function II

- The computation of the expectation (a) in (8):
- Fix T. Let

$$g_t = \exp\left(\kappa \int_0^t h(u, T-s)\langle \boldsymbol{\vartheta}, Z_s\rangle ds\right), \quad 0 < t \leq T \tag{9}$$

$$G_t := g_t Z_t. \tag{10}$$

- Then

$$\begin{aligned} dG_t &= g_t dZ_t + Z_t dg_t \\ &= g_t(Q' Z_t dt + dM_t) + Z_t g_t \kappa h(u, T-t)\langle \boldsymbol{\vartheta}, Z_t\rangle dt \\ &= \left(Q' + \kappa h(u, T-t)\Theta\right) g_t Z_t dt + g_t dM_t \end{aligned} \tag{11}$$

- Consider

$$\hat{G}_t = \mathbb{E}[G_t | Z_0 = Z] = \Phi(s,t)Z.$$

Matrix ODE

- Along the lines of Elliott et al. [2016], we showed that since

$$\mathbb{E}[G_t] = \langle \hat{G}_t, \mathbf{1} \rangle$$

- Then

$$E\left[\exp\left(\kappa \int_0^T h(u, T-s)\langle \boldsymbol{\vartheta}, Z_s \rangle ds\right)\right] = \langle \Phi(0,T) Z_0, \mathbf{1} \rangle,$$

- where Φ is a solution of the matrix ODE given by

$$\frac{d\Phi(s,t)}{dt} = \left(Q' + \kappa h(u, T-t)\Theta\right) \Phi(s,t), \; s \leq t, \; \text{with} \; \Phi(s,s) = \mathbf{I}, \quad (12)$$

- where $\Theta = \text{diag}[\boldsymbol{\vartheta}]$.

Semi-Analytic Pricing Formula

- To price options, we use the well-known Fourier-inversion formula of Gil-Pelaez [1951] which leads to a semi-analytic closed-form solution given by:

$$C_0 = e^{-rT}\mathbb{E}\left[(e^X - K)^+\right] = \mathbb{E}[e^X]\Pi_1 - e^{-rT}K\Pi_2, \tag{13}$$

where the probability quantities Π_1 and Π_2 are given by:

$$\begin{aligned}\Pi_1 &= \mathbb{E}[e^X \mathbb{I}_{\{e^X>K\}}]/\mathbb{E}[e^X] = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \text{Re}\left[\frac{e^{-\mathrm{i}u\log(K)}\varphi_X(u-i)}{\mathrm{i}u\varphi_X(-\mathrm{i})}dz\right] \\ \Pi_2 &= \mathbb{P}\{e^X > K\} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \text{Re}\left[\frac{e^{-\mathrm{i}u\log(K)}\varphi_X(u)}{\mathrm{i}u}dz\right].\end{aligned} \tag{14}$$

Monte Carlo Approaches

- Two Monte Carlo simulations have been implemented:
  1. Full Monte Carlo
     - We first simulate the 3-dimensional process (B, W, θ).
     - The option pay-out can be obtained (in the risk neutral world) in each simulation.
  2. Partial Monte Carlo
     - This is a novel method in this context
     - We simulate the paths of θ_s and then evaluate for each realization $\theta_s(\omega)$, the formula (8), i.e., for $l = 1, 2, \cdots, N$

$$E[e^{\mathrm{i}uX_t}](\omega_l) = \exp\left(\int_0^t h(u, T-s)\left(\kappa\theta_s(\omega_l) + \frac{V_0 s^{-\alpha}}{\Gamma(1-\alpha)}\right) ds\right), \quad (15)$$

     - Then

$$E[e^{\mathrm{i}uX_t}] \sim \frac{1}{N}\sum_{l=1}^{N} E[e^{\mathrm{i}uX_t}](\omega_l) \quad (16)$$

The Pricing Parameters

Table: Adopted from Elliott et al. [2016]

Parameters	value
$S(0)$	100
r	0.05
σ	0.4
ρ	-0.5
κ	3
$\theta_0 = [\theta^1 \ \theta^2]$	$[0.025 \ 0.075]$
Q_E	$\begin{bmatrix} -1 & 1 \\ 0.5 & -0.5 \end{bmatrix}$
No. of Simulations	$1\ 000\ 000$
Time Steps	250

Call prices, $v_0 = 0.02 < \theta^1 < \theta^2, H = 0.5$

(a) Starting in a low state: $\theta_0 = \theta^1$

K/T	0.25			0.5			0.75		
	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic
	Price	std Error	Fourier	Price	std Error	Fourier	Price	std Error	Fourier
90	11.57148	0.00745	11.48849	13.27125	0.01006	13.25924	15.49823	0.01392	15.01218
95	7.38753	0.00648	7.17254	9.59107	0.00929	9.27515	10.73807	0.01015	11.24349
100	3.97505	0.00504	3.62547	6.68191	0.00842	5.88019	7.39669	0.00878	7.95919
105	1.69219	0.00338	1.33534	3.66323	0.00604	3.29490	4.71588	0.00721	5.28047
110	0.55198	0.00192	0.35727	1.83362	0.00427	1.62772	2.73756	0.00558	3.27974
115	0.14608	0.00098	0.08044	0.63928	0.00244	0.73575	1.71990	0.00460	1.92863
120	0.03219	0.00045	0.01697	0.35077	0.00187	0.31933	1.76078	0.00526	1.09545

(b) Starting in a high state: $\theta_0 = \theta^2$

K/T	0.25			0.5			0.75		
	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic
	Price	std Error	Fourier	Price	std Error	Fourier	Price	std Error	Fourier
90	11.57717	0.00729	11.73234	13.71997	0.01122	13.87580	15.72714	0.01448	15.84949
95	7.36691	0.00633	7.63948	9.94275	0.00999	10.17181	12.48412	0.01384	12.34856
100	3.90727	0.00489	4.31660	6.73111	0.00851	7.02647	9.49103	0.01238	9.29809
105	1.60708	0.00323	2.02691	4.20529	0.00688	4.53709	6.93522	0.01078	6.74947
110	0.49681	0.00179	0.77241	2.74969	0.00587	2.72878	4.87362	0.00915	4.71991
115	0.12359	0.00089	0.24232	1.06738	0.00340	1.53220	3.22032	0.00740	3.18403
120	0.02729	0.00041	0.06548	0.60658	0.00264	0.80856	2.16359	0.00611	2.07808

Call prices, $v_0 = 0.02 < \theta^1 < \theta^2, H = 0.1$

(a) Starting in a low state: $\theta_0 = \theta^1$

K/T	0.25				0.5				1			
	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic
	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier
80	21.21834	0.08142	21.04928	20.99613	22.70236	0.10612	22.39585	21.92995	24.95195	0.14809	24.43662	24.54135
85	16.53185	0.07505	16.79874	16.89825	18.25055	0.10175	18.15340	17.71934	20.87358	0.14293	20.32966	20.28785
90	11.98653	0.06997	12.31647	11.94876	13.60705	0.09478	13.86663	13.37147	16.74382	0.13051	16.48881	16.32680
95	7.73380	0.06096	7.62077	6.67428	9.73972	0.08340	9.91929	8.89896	12.74631	0.11821	12.76987	12.64163
100	3.81818	0.04910	3.78302	2.46191	6.02765	0.07292	6.42048	4.93519	9.40263	0.11208	9.42984	9.30234
105	1.44438	0.03752	1.51491	0.24732	2.96650	0.05546	3.80160	2.15772	6.43873	0.09919	6.60637	6.46152
110	0.42772	0.02180	0.39556	0.12512	1.40261	0.04262	2.06220	0.75938	3.86032	0.08174	4.35670	4.24546

(b) Starting in a high state: $\theta_0 = \theta^2$

K/T	0.25				0.5				1			
	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic
	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier
80	21.22925	0.07501	20.98623	20.96890	22.79630	0.11497	22.15238	22.14655	25.74824	0.05678	25.50586	24.97705
85	16.52081	0.07119	16.69764	16.75793	18.18483	0.10556	17.86022	17.80336	21.70714	0.05376	21.55210	20.86047
90	11.96638	0.06593	12.11152	12.04218	13.99007	0.09802	13.93820	13.62524	17.96501	0.05086	17.89002	17.03892
95	7.50552	0.05791	7.50617	7.16747	9.84658	0.08937	10.07705	9.61300	14.15027	0.04681	14.58459	13.55432
100	3.63218	0.04688	3.79966	3.17796	6.30056	0.07959	6.45585	6.06377	11.00267	0.04286	11.63212	10.45512
105	1.19131	0.03641	1.33402	0.84507	3.44352	0.06389	3.79153	3.35864	8.10423	0.03822	9.08070	7.79629
110	0.38912	0.02242	0.51241	0.12238	1.70127	0.05041	2.05766	1.65555	5.76181	0.03393	6.93307	5.61828

Call prices, $v_0 = 0.02 < \theta^1 < \theta^2, H = 0.1, Q \succ Q_E$

(a) Starting in a low state: $\theta_0 = \theta^1$

K/T	0.25				0.5				1			
	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic
	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier
80	21.47068	0.07908	21.00175	20.98273	22.63093	0.10858	22.28225	22.01076	25.42472	0.15965	25.79463	24.70909
85	16.66761	0.07313	16.70332	16.84830	17.97404	0.10169	18.06484	17.72671	21.14268	0.14998	20.03459	20.48775
90	11.80479	0.06609	12.27504	11.98113	13.90196	0.09522	13.89394	13.45825	17.08855	0.14001	16.49172	16.56613
95	7.59101	0.05911	7.61382	6.84312	9.68327	0.08705	9.99224	9.19971	13.44172	0.13147	13.35790	12.97795
100	3.77509	0.04935	3.78298	2.70477	6.05559	0.07620	6.41210	5.42301	10.14287	0.12180	10.23271	9.77358
105	1.24446	0.03634	1.39731	0.44667	3.19847	0.06080	3.79761	2.66488	6.97217	0.10751	7.34930	7.03425
110	0.46183	0.02466	0.50095	0.04683	1.50331	0.04772	2.08867	1.11439	4.69907	0.08864	5.18584	4.82989

(b) Starting in a high state: $\theta_0 = \theta^2$

K/T	0.25				0.5				1			
	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic	Monte Carlo			Semi-Analytic
	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier	Full	std Error	Partial	Fourier
80	21.27567	0.07903	21.07213	20.97415	22.70422	0.11488	22.25050	22.04900	25.71770	0.16603	24.45007	24.74451
85	16.51986	0.07355	16.78900	16.79487	18.38982	0.10809	17.98040	17.73785	21.25906	0.15787	21.93313	20.53618
90	12.02166	0.06611	12.01867	12.00798	13.84454	0.09795	13.84289	13.49585	17.24327	0.14703	17.88039	16.62739
95	7.57394	0.05978	7.72731	7.01520	9.88294	0.08755	9.86080	9.31996	13.34188	0.13406	13.78744	13.05632
100	3.69629	0.04933	3.79923	2.96032	6.05455	0.07464	6.43077	5.62394	10.13329	0.12298	10.44554	9.87387
105	1.31239	0.03849	1.47665	0.66300	3.50841	0.06450	3.87000	2.88394	7.42620	0.10936	7.83990	7.15294
110	0.53143	0.02878	0.42124	0.04452	1.72727	0.04967	2.07582	1.28172	5.01555	0.09459	5.83673	4.95390

Call prices - Heston model modifications, $T = 1$

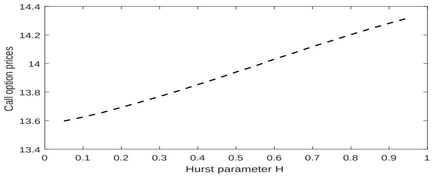
(a) Starting in a low state: $\theta_0 = \theta^1$

Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic
Price	std Error	Fourier	Price	std Error	Fourier	Price	std Error	Fourier
24.18566	0.00023	24.40145	24.87668	0.12432	24.41194	24.95195	0.14809	24.54135
19.73528	0.00033	20.01877	20.64484	0.11906	20.03026	20.87358	0.14293	20.28785
15.54209	0.00042	15.86069	15.91675	0.10802	15.87262	16.74382	0.13051	16.32680
11.74930	0.00050	12.02694	12.01290	0.10022	12.03865	12.74631	0.11821	12.64163
8.50614	0.00055	8.63622	8.28652	0.08782	8.64671	9.40263	0.11208	9.30234
5.91152	0.00058	5.80791	5.03983	0.07613	5.81549	6.43873	0.09919	6.46152
3.97837	0.00058	3.62625	2.88119	0.06492	3.62874	3.86032	0.08174	4.24546
Classical Heston			Rough Heston			Regime Switching Rough Heston		

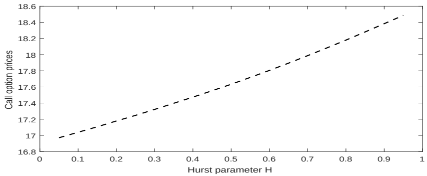
(b) Starting in a high state: $\theta_0 = \theta^2$

Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic	Monte Carlo		Semi-Analytic
Price	std Error	Fourier	Price	std Error	Fourier	Price	std Error	Fourier
25.24961	0.00025	25.56759	26.12745	0.18946	25.64066	25.74824	0.05678	24.97705
21.34310	0.00028	21.66156	22.12496	0.18115	21.74460	21.70714	0.05376	20.86047
17.77426	0.00030	18.04638	17.86913	0.16392	18.13614	17.96501	0.05086	17.03892
14.58986	0.00032	14.76582	14.37190	0.15299	14.85821	14.15027	0.04681	13.55432
11.81316	0.00034	11.85344	11.37817	0.13983	11.94405	11.00267	0.04286	10.45512
9.44701	0.00035	9.32861	8.53524	0.12659	9.41330	8.10423	0.03822	7.79629
7.47475	0.00035	7.19422	6.24291	0.11296	7.26965	5.76181	0.03393	5.61828
Classical Heston			Rough Heston			Regime Switching Rough Heston		

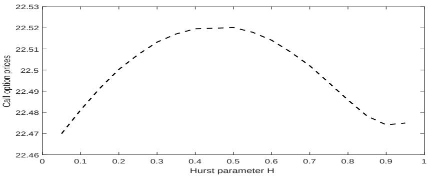
Hurst Parameter Price Impact



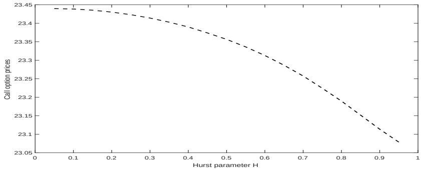
(a) 6 months maturity



(b) 1 year maturity

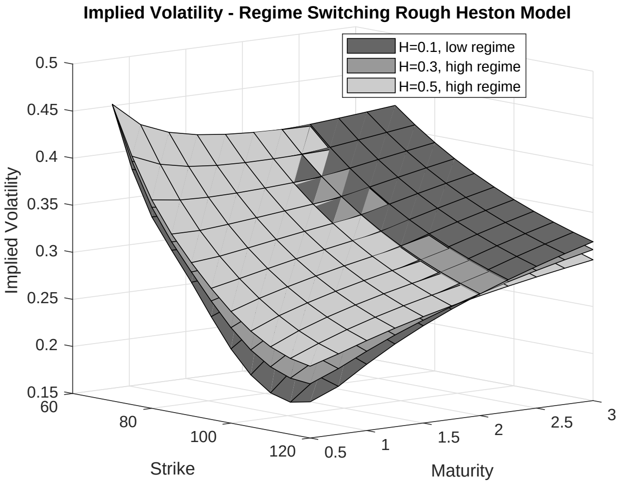


(c) 1.85 years maturity



(d) 2.5 years maturity

Implied Volatility Surface



Computational Speed

	Pricing Method		
Model	Semi-Analytic	Full Monte Carlo	Partial Monte Carlo
Classical Heston	0.534319	8.385977	-
Regime Switching Heston	5.065121	30.461079	-
Rough Heston	4.041738	837.789867	-
Regime Switching Rough Heston	19.56441	823.823312	14.701957

*Time measured in seconds.

Concluding Remarks

- We studied the regime switching rough Heston models.
- The main goal is to construct a unified framework that captures the stylized facts of volatility.
- We developed a pricing engine and fully implemented this analytic approach
- We benchmark these semi-analytic prices against two types of Monte-Carlo-simulations.
- Talk is based on the paper with Mesias Alfeus and Erik Schloegl: "Regime switching rough Heston model" in Journal of Futures Markets Volume 39, Issue 5. 2019

Thank you!

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