Some Insurance Valuation and Design Problems with Aggregate Risk

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OVERVIEW

Standard insurance valuation/design problems

- Pooling homogeneous, (conditionally) independent risks
- Representative agent/policyholder
- If portfolio is large, only aggregate risk matters

In practice, however...

- Aggregate risk can arise endogenously (e.g., policyholder behavior)
- Valuation and contract design should internalize aggregate risk

Some interesting problems

- Optionality in long term insurance contracts
  - Ex-ante i.i.d. risks give rise to endogenous aggregate risk
- P&C examples
  - Conditionally i.i.d. risks and coverage for high layers of exposure
  - Multi-year agricultural insurance in supply chain risk management
Overview

Optionality in Life insurance

Testing for Dynamic Adverse Selection

P&C Applications

Conclusion
OPTIONALITY IN LIFE INSURANCE

Long term insurance contracts

- Longevity/mortality risk assessment: is it enough?
- Are financial and demographic risk factors uncorrelated?
- Asset Management Charges (AMCs) vs. level premiums
- Role of contract design and policyholder behavior
- Endogenous dependence and aggregate risk via optionality

Policyholder behavior

- ‘Rational’ exercise of options
- Testing for dynamics adverse selection
- Making sense of actuarial approaches: **pricing basis** & and **lapse/surrender basis**
SETUP

Longevity risk

• Aggregate changes in survival probabilities
• Both aggregate and idiosyncratic risk relevant in the presence of optionality

Reference setup: conditionally Poisson / Cox setting (more generally, see Tappe and Weber, 2014)

• At contract inception (time 0), portfolio of insureds with death times $\tau^1, \ldots, \tau^n$
• Each $\tau^i$ has force of mortality $\mu^i(t)$
• Possible representations: $\mu^i(t) = X(t) + Y^i(t)$ or $\mu^i(t) = X(t) Y^i(t)$

Portfolio vs. population

• Surrender/lapse time $\theta^i$
• Exit from the portfolio at stopping time $\sigma^i := \tau^i \land \theta^i$
POLICYHOLDER BEHAVIOR

Value of the contract to insured $i$ is

$$v^i(t; \sigma^i, c) = 1_{\sigma^i > t} \mathbb{E}_Q^i \left[ \int_t^{\theta^i \wedge T} e^{-\int_t^s (r(u) + \mu^i(u)) du} dG^i(s; c) \left| \mathcal{F}_t \right. \right].$$

- $G^i(t; c)$: cumulative gains to the insured from holding the insurance contract, with $c \in C$ contract configuration (including guarantees)
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- $G^i(t; c)$: cumulative gains to the insured from holding the insurance contract, with $c \in C$ contract configuration (including guarantees)

Some issues...

- $Q^i$ private valuation of insured $i$
- $\mathbb{F}^i := (\mathcal{F}^i_t)_{t \geq 0}$ (private) information available to insured $i$
- Endogenous $\sigma^i$ (optimal stopping problem $\theta^i$)
  - More generally, one should also allow for other dimensions of optionality (fund switches, partial withdrawals, etc.)

Question: how to proxy for $v^i$ across p/h’s?
DYNAMIC ADVERSE SELECTION

Individuals **ex-ante** identical

- **At contract inception (time 0)** policyholders' death times $\tau_1, \ldots, \tau^n$ have (say) independent intensities $\mu^i_1, \ldots, \mu^n_i$ with the **same law as process $\mu$**
- $(F(t))_{t \geq 0}$ vector of financial risk factors (say) independent of mortality
DYNAMIC ADVERSE SELECTION

Individuals \textbf{ex-ante} identical

- At contract inception (time 0) policyholders' death times $\tau^1, \ldots, \tau^n$ have (say) independent intensities $\mu^1, \ldots, \mu^n$ with the \textit{same law} as process $\mu$
- $(F(t))_{t \geq 0}$ vector of financial risk factors (say) independent of mortality

\textbf{Ex-post} mortality profile of the portfolio

- Different trajectories $(\mu^i(t, \omega_1), F(t, \omega_1))_{t \geq 0}, \ldots, (\mu^i(t, \omega_k), F(t, \omega_k))_{t \geq 0}$ make staying in the contract more or less valuable for p/h $i$
- The moneyness of any guarantee/option is at shaped at least by $\mu^i$ and $c \in C$ (contract design channel)
- \textbf{Portfolio mortality} (average intensity)

$$\bar{\mu}_p(t) := \frac{\sum_{i=1}^{n} \mu^i(t) 1_{\sigma^i > t}}{\sum_{i=1}^{n} 1_{\sigma^i > t}}.$$  

- The insurer cannot observe $\mu^i$, but can try to recover the \textit{law} of $\bar{\mu}_p$ based on $c \in C$ and relevant (observable) state variables
FRAILTY REPRESENTATION

Change in intensity process

- Think of death times $\tau$ (representative member of the population) and $\bar{\tau}_p$ (average portfolio member)

- Dynamic frailty representation: individual (on $\{\sigma^i > t\}$) or average/representative portfolio member (on $\{\sigma^{(n)} > t\}$)

$$
\mu^i(t) = \mu(t)\eta^i(t; c) \quad \bar{\mu}_p(t) = \mu(t)\bar{\eta}(t; c)
$$

with $(\eta^i(t, c))_{t \geq 0} > 0$ and $(\bar{\eta}(t; c))_{t \geq 0} > 0$ dynamic frailty processes; under suitable assumptions, the Cox setting is preserved (e.g., Biffis, Denuit, Devolder, 2010)

- Think of change in intensity as captured by a suitable change of probability measure: likelihood ratio driven by dynamic frailty process
Pricing

Insurer’s view

- Baseline reference probability measures $Q_F$ (financial factors) and $P_M$ (population mortality)
- Pricing with $Q := Q_F \otimes P_M$ (wrong!)

$$V^i(0; \theta^i, c) = V(0; \theta, c) = \mathbb{E}^Q \left[ \int_0^{\theta \wedge T} e^{-\int_0^s (r(u) + \mu(u))\,du} \, dG(s; c) \right].$$
PRICING

Insurer’s view

• Baseline reference probability measures $\mathbb{Q}_F$ (financial factors) and $\mathbb{P}_M$ (population mortality)

• Pricing with $\mathbb{Q}_F$ (reflects portfolio mortality)

$$V_p^i(0; \theta^i, c) = V_p(0; \theta, c) = \mathbb{E}^{\mathbb{Q}_F} \left[ \int_{0}^{\theta \wedge T} e^{-\int_{0}^{s} (r(u) + \mu_p(u))du} dG(s; c) \right].$$

• The representative policyholder’s death time is $\tau_p$ and not $\tau$...


**Pricing**

**Insurer’s view**

- Baseline reference probability measures $\mathbb{Q}_F$ (financial factors) and $\mathbb{P}_M$ (population mortality)
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\[
V_p^i(0; \theta^i, c) = V_p(0; \theta, c) = \mathbb{E}^{\mathbb{Q}_p} \left[ \int_0^{T_{\theta}} e^{-\int_0^s (r(u) + \mu_p(u)) du} dG(s; c) \right].
\]

- The representative policyholder’s death time is $\tau_p$ and not $\tau$...

**Implications**

- Change in intensity and no factorization in general even if mortality and financial risk factors uncorrelated
- Surrender/lapse basis jointly determined with mortality basis
- Useful framework for contract design: optimize with respect to $c \in C$
  - Determine fair AMCs
  - Steer the portfolio toward a target mortality risk profile
EXAMPLES

Baseline example

- 20-year VA contract
- 45 male, non smoker
- GMAB (accumulation): 2.5% p.a.
- GMSB (survival): premiums paid with 0% or 2.5% p.a. guarantee; but surrender penalties in the first 5 years of contract
- GMDB (death): varying from zero to $2 \times \text{GMAB guaranteed rate}$
- Reference fund: Geometric Brownian Motion, 15% volatility

GMWB (withdrawal) and GMLB (lifetime) also interesting...

- Wedge between systematic and idiosyncratic risk more important
AVERAGE FRAILTY (GMSB: premium paid)

Source: Benedetti and Biffis (2016).
AVERAGE FRAILTY (GMSB: premium paid rolled over at 2.5% p.a.)

Source: Benedetti and Biffis (2016).
FAIR AMCs (GMSB: initial amount paid into the policy)

Source: Benedetti and Biffis (2016).
FAIR AMCs (GMSB: initial amount rolled over at 2.5% p.a.)

Source: Benedetti and Biffis (2016).
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Possible approaches suggested by our framework

- Use frailty process \((\eta(t; c))_{t \geq 0}\)
- Use ‘distance’ between \(\mu(t)\) and \(\bar{\mu}_p(t)\)
- Use ‘distance’ between (conditional) law of \(\tau\) and \(\bar{\tau}_p\)
TESTING FOR DYNAMIC ADVERSE SELECTION

Possible approaches suggested by our framework

- Use frailty process \((\eta(t; c))_{t \geq 0}\)
- Use ‘distance’ between \(\mu(t)\) and \(\mu_p(t)\)
- Use ‘distance’ between (conditional) law of \(\tau\) and \(\tau_p\)

A class of divergences (e.g., Vonta-Karagrigoriou, 2010)

\[
D_{\tau, \tau_p}^\psi(t) = \int_t^T \psi \left( \frac{dP(t < \tau_p \leq s | F_t)}{dP(t < \tau \leq s | F_t)} \right) dP(t < \tau \leq s | F_t),
\]

with \(\psi \in C^2(\mathbb{R}_+; \mathbb{R}), \psi(1) = 0\)

- Examples: \(\alpha\)-divergences (Csiszár’s family), Kullback-Leibler, Hellinger, etc.
- Different from standard approaches (e.g, Albert et al., 1999; He, 2011)

\[
\text{Actual_deaths}_t / \text{Expected_deaths}_t = \alpha + \beta \times \text{Lapse}_t + \varepsilon
\]

\[
P(lapse_i = 1) = F(a + b \times \text{health_shock}_i)
\]
### SOME RESULTS

\[ \beta \text{ estimates for regressions } 
\begin{align*} 
  y_{t+1} &= \alpha + \beta \times \text{lapse\_ratio}_t + \varepsilon_t. \\
  y_{t+1} &= \eta y_{t+1} = KL(\mu, \mu_p) 
\end{align*} \]

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Source: Benedetti and Biffis (2016).

\[ \beta \text{ estimates for regressions } 
\begin{align*} 
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Source: Benedetti and Biffis (2016).

- Simulated environment for 2500 traditional contracts issued to male non-smokers aged 50.
- Maturity \(T = 20\) years, decreasing surrender penalties during the first 3 years of contract. Death (\(D\)) and survival (\(S\)) benefits.
- Use average frailty \(\bar{\eta} = \mu_p/\mu\) as proxy for actual/expected deaths.
RISK SHARING AND LIMITED LIABILITY

A risk sharing problem (Arrow/Raviv) with limited liability

- One-period model with a continuum of insurees modeled as the measure space $(M, M, \mu)$ of the unit interval $M = [0, 1]$, with $\mu(M) = 1$.
- Insurer maximizes function $V$ over indemnities $(I_i)$, and risky asset allocation $(\alpha)$

$$V(\alpha, (I_i)) = \max \left\{ \left( A + \int_0^1 \pi_i \mu(di) \right) (1 + \alpha R) - \int_0^1 I_i(X_i) \mu(di), 0 \right\}$$

where $I_i(X_i)$ is indemnity for p/h $i$’s loss $X_i$ financed by insurance premium $\pi_i \geq 0$

- Can optimize relative to initial capital $A$
- Can add regulatory constraints

- Each insuree satisfies the participation constraint

$$E \left[ u_i(w_i - \pi_i - X_i + I_i(X_i)1_{D=0} + \gamma I_i(X_i)1_{D=1}) \right] \geq u_i,$$

with $\{D = 1\}$ default event, $\gamma \in [0, 1]$ recovery rate
AGGREGATION

Assume $X_i = Y_i + Z$ for all $i \in [0, 1]$

- $(Y_i)$ essentially uncorrelated (and i.d. for simplicity here), $(Y_i), Z \in L^2$
- Use Sun (2006)’s Exact Law of Large Numbers.

Some special cases

- Idiosyncratic risk only ($Z = 0$)
  $$\int_0^1 I(X_i) \mu(d\xi) = \int_0^1 E[I(X_i)] \mu(d\xi) = E[I(X_i)] = E[I(X)] \text{ a.s.}$$

- Systematic risk only ($Y_i = 0$): some examples to follow
  $$\int_0^1 I(X_i) \mu(d\xi) = \int_0^1 E[I(X_i)|Z] \mu(d\xi) = E[I(X_i)|Z] \ldots$$

- Good model lies somewhere in the middle
OPTIMAL INDEMNITY SCHEDULE

Source: Biffis and Millossovich (2013).
OPTIMAL RETENTION LEVELS

Source: Biffis and Millossovitch (2013).
Average retention levels in US P&C, evidence from reinsurance purchases. Source: Guy Carpenter (e.g., Froot 1997, 2001).
Source: Biffis and Millossovich (2013).
SUPPLY CHAIN RISK MANAGEMENT

General questions

- How to unlock value in supply chains via risk sharing arrangements?
- How to build inclusive and resilient local-to-global supply chains?

Agricultural insurance example (World Food Program)

- Farmers organizations as aggregators of small farmholders
- Banks as providers of credit (better inputs and technology)
- Agro-dealers as off-takers
- (Re)insurers cover extreme crop yield losses

Challenges (World Food Program)

- How to incentivize farmers to switch to more resilient production technologies?
- Technology takes time to demonstrate its value (several harvesting seasons)
- At odds with short term contracts offered by (re)insurers
Effect of Return Period on Production Loss

- Rainfed maize
- Short cycle maize
- Irrigated maize

Source: Biffis and Chavez (2016).
MULTI-YEAR PROGRAMS

Insurance contract: **structure and payouts**

- **Payout**
  - Volume of deficit production at average production at \( p^* \) price

- **Regional weather index**
  - Increasing severity and loss, decreasing probability

- **Average/normal prices**

- **Loan defaults**
- **Weather driven production losses**
- **Upward price pressure**

Source: WINnERS project, Biffis and Chavez (2016).

- Uncertainty in medium-to-long-term climate projections is source of aggregate risk
- Explicitly allow for random fraction (\( Q \)) of farmholders affected by crop yield losses
- Optimal contract \( I^*(X, Q) \) entails contingent attachment/detachment points (Biffis and Louaas, 2016)
MULTI-YEAR PROGRAMS

Insurance contract: indirect insurance for farmer

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- Explicitly allow for random fraction \((Q)\) of farmholders affected by crop yield losses
- Optimal contract \(I^*(X, Q)\) entails contingent attachment/detachment points (Biffis and Louaas, 2016)

Source: WINnERS project, Biffis and Chavez (2016).
CONCLUSION

Standard valuation/risk sharing models useful
- Risk pooling (predictability, vanishing cost of capital)
- Representative policyholder approach

Allowing explicitly for aggregate risk can be more useful
- From idiosyncratic risk to systematic risk via optionality
- Systematic risk, aggregate risk, and counterparty risk
- New avenues for risk sharing via complete contracts

Technical caveats
- Some interesting challenges: incomplete market valuation methods and feedback effects, existence and uniqueness of solutions in risk sharing problems, etc.
THANK YOU