Comparing Life Insurer Longevity Risk Transfer Strategies in a Multi-Period Valuation Framework

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Motivation

- Increased interest in reinsurance and longevity bonds to manage **longevity risk** for products that guarantee a retirement income (life annuities, pensions)
- Longevity risk management strategies
  - Capital and product pricing under different solvency regimes Nirmalendran *et al.* (2012)
  - Reinsurance (Olivieri, 2005; Olivieri and Pitacco, 2008; Levantesi and Menzietti, 2008)
  - Securitization (Cowley and Cummins, 2005; Wills and Sherris, 2010; Biffis and Blake, 2010; Gupta and Wang, 2011)
- Each strategy involves differing costs and risks
- Research Question: How do longevity risk management decisions impact the firm's value for an insurer issuing life annuities allowing for frictional costs, market premiums, and solvency?

Introduction

- **Investigate the impact of longevity risk transfer strategies on an insurer's solvency and shareholder value for an annuity portfolio.**
- A multi-period valuation framework: one of the main contributions of the paper, allows for
  - The costs of transferring longevity risk.
  - Regulatory capital requirements and capital relief.
  - Cost of holding capital.
  - Financial distress costs.
  - Policyholders' price-default-demand elasticity.
- Analyze the interaction between capital management and reinsurance or securitization.
- Valuation approaches
  - Economic Balance Sheet (EBS)
  - Market-Consistent Embedded Value (MCEV)

Introduction

- Stochastic mortality model with both systematic and idiosyncratic longevity risk.
- Risk transfer strategies
  - Reinsurance: indemnity-based, covers both systematic and idiosyncratic longevity risk.
  - Securitization: index-based, covers only systematic longevity risk.
- Solvency capital requirements - Solvency II.
- **Results:** Longevity risk management strategies...
  - reduce the insurer's default probability.
  - increase shareholder value and,
  - reduce the volatility of the shareholder value.
  - reduce the level and the volatility of frictional costs.
  - reduce investor uncertainty.

Affine Mortality Model

- Stochastic mortality model by Blackburn and Sherris (2012).
  - Based on forward (cohort) mortality rates
  - Avoids need for nested simulations at future time points when valuing future liabilities.
  - Model structure: HJM forward rate models (Heath *et al.*, 1992).
  - Model gives stochastic forward interest rates and forward mortality rates.
- Use a model variant with 2-stochastic mortality risk factors, a deterministic volatility function and Gaussian dynamics (Blackburn, 2013).
- Model is calibrated to Australian male population ages 50-100, years 1965-2007

Pricing Measure

- Risk-neutral measure: best estimate cohort survivor curve, used to value annuity cash flows without loading.
- Pricing and market valuation measure: construct a new martingale measure.
  - λ: constant price of risk: instantaneous Sharpe ratio (Milevsky and Promislow, 2001).
  - No impact on the volatility function, but scaling of the initial forward mortality curve.
  - Calibrate from quoted reinsurance loadings (survivor swap premium): $\lambda = 0.1555$
- Assume interest and mortality rates are independent.
- Assume interest rates are deterministic.

Pricing Measure

- Best estimate and market pricing survivor curves with 99% confidence intervals.

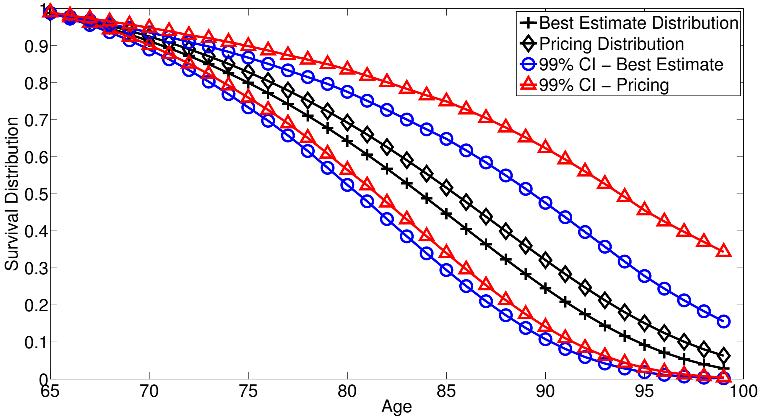


Figure: Cohort Survival Distribution Aged 65 in 2010

Framework

- Monte-Carlo simulation of an insurer with an annuity portfolio
  - Portfolio run-off from ages 65 to 100
  - Annuity demand related to premium loading and default probability
- Idiosyncratic risk due to portfolio size
- Risk transfer (static hedge) through:
  - Survivor Swap - indemnity based
  - Survivor Bond - index based
- Risk transfer options
  - 50% or 100% risk transfer
  - 50% or 100% capital relief
- Mark-to-Market valuation of liabilities / reserve
- EBS and MCEV balance sheet items
  - Frictional costs due to holding capital
  - Recapitalization costs
  - Excess capital distributed as dividends
  - Initial shareholder capital
  - Expenses

Monte Carlo Simulation and Idiosyncratic Longevity Risk

- Implement the mortality model as a discrete time version of the HJM model.
- Use Monte Carlo simulation based on Glasserman (2003).
- For each simulation path m:
  - Generate mortality rates to give a survivor index.
  - Generate forward mortality curves for each discrete time point t_i.
  - Expected number of survivors: $\widehat{l}^{(m)}(t_i; x)$
- Idiosyncratic longevity risk:
  - Random death times for individuals: the first time the implied force of mortality for path m is above ϱ.
  - ϱ is an exponential random variable with parameter 1.
  - Gives the actual number of survivors: $\widetilde{l}^{(m)}(t_i; x)$

Idiosyncratic Longevity Risk

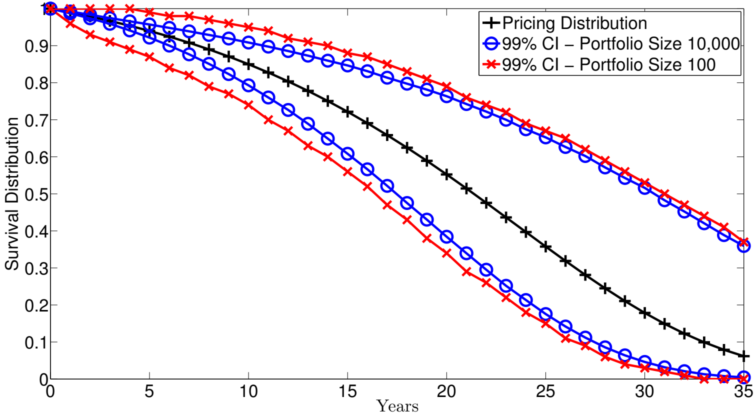


Figure: Portfolio Survivors $\widetilde{l}^{(m)}(t;x)$

Annuity Pricing and Reserving

- The **market value** of an annuity that pays \$$b$ per year to each annuitant in a cohort age x at time-0 is

$$\widehat{a}(0,t_n;x)=\sum_{t_s=t_1}^{t_n} b\cdot\exp\Big(-\sum_{t_j=t_0}^{t_{s-1}}\big(\widehat{f}(0,t_j)+\widehat{\mu}(0,t_j;x)\cdot[t_{j+1}-t_j]\big)\Big).$$

- The path dependent **forward market value** of an annuity is

$$\widehat{a}(0,t_i,t_n;x)=\sum_{t_s=t_{i+1}}^{t_n} b\cdot\exp\Big(-\sum_{t_j=t_0}^{t_{s-1}}\big(\widehat{f}(0,t_j)+\widehat{\mu}(0,t_j;x)\cdot[t_{j+1}-t_j]\big)\Big).$$

- The **fair value** of an annuity that pays \$$b$ per year to each annuitant in a cohort age x at time-0 is

$$\widehat{\overline{a}}(0,t_n;x)=\sum_{t_s=t_1}^{t_n} b\cdot\exp\Big(-\sum_{t_j=t_0}^{t_{s-1}}\big(\widehat{f}(0,t_j)+\widehat{\overline{\mu}}(0,t_j;x)\big)\cdot[t_{j+1}-t_j]\Big),$$

where $\widehat{\overline{\mu}}(0,t_j;x)$ is the best estimate cohort forward survivor curve.

Portfolio

- Annuity single premium, γ^P - premium loading

$$\pi = b \cdot \left(1 + \gamma^P\right) \cdot \widehat{\overline{a}}(0, t_n; x).$$

- Market reserve - unhedged

$$\widetilde{V}_p^{(m)}(t_i; x) = \widetilde{I}^{(m)}(t_i; x) \cdot \widehat{a}^{(m)}(t_i; x).$$

- Market reserve - hedged

$$\widehat{V}_h(t_i; x) = n_0 \cdot \widehat{S}(0, t_i; x) \cdot \widehat{a}(0, t_i, t_n; x).$$

- **Total portfolio reserve**

$$\widetilde{V_s}^{(m)}(t_i; x) = (1 - \omega_h)\widetilde{V}_p^{(m)}(t_i; x) + \omega_h \widehat{V}_h(t_i; x).$$

Portfolio

- Solvency Capital Requirement - $\phi = 0.2$

$$\widetilde{M}_p^{(m)}(t_i) = \widetilde{V}_p^{(m)}(t_i)|\texttt{Longevityshock} - \widetilde{V}_p^{(m)}(t_i)$$

- Total SCR, assuming ω_c, is the proportion of hedged liabilities that are given capital relief.

$$\widetilde{M}_h^{(m)}(t_i) = \widetilde{M}_p^{(m)}(t_i) \cdot (1 - \omega_c).$$

- **Total Reserve**

$$\widetilde{V}^{(m)}(t_i) = \widetilde{V}_s^{(m)}(t_i) + \widetilde{M}^{(m)}(t_i) + \widetilde{V}_e^{(m)}(t_i) \tag{1}$$

Cash Flows

- No hedging

$$\widetilde{CF}^{(m)}(t_i) = -b \cdot \widetilde{I}^{(m)}(t_i; x) - \widetilde{E}^{(m)}(t_i).$$

- Survivor Swap

$$= -b \cdot \left[\widetilde{I}^{(m)}(t_i; x) + \omega_h \Big((1 + \gamma^R) \cdot \widehat{\overline{S}}(0, t_i; x) - \widetilde{I}^{(m)}(t_i; x) \Big) \right] - \widetilde{E}^{(m)}(t_i).$$

- Survivor Bond

$$= -b \cdot \left[\widetilde{I}^{(m)}(t_i; x) + \omega_h \Big((1 + \gamma^R) \cdot \widehat{\overline{S}}(0, t_i; x) - \widehat{I}^{(m)}(t_i; x) \Big) \right] - \widetilde{E}^{(m)}(t_i).$$

Solvency, Dividends, and Recapitalization

- $\widetilde{A}^{(m)}(t_i) < \widetilde{V}_s^{(m)}(t_i)$: there are insufficient assets to cover time-t liabilities and the insurer **defaults.**
  - Annuitants receive only the residual assets
  - Limited Liability Put Option:
    $\widetilde{LLPO}^{(m)}(t_i) = \max\{0, \widetilde{V}_s^{(m)}(t_i) - \widetilde{A}^{(m)}(t_i)\}$.

- $\widetilde{A}^{(m)}(t_i) - \widetilde{V}^{(m)}(t_i) < 0$: no default, but insufficient capital to meet regulatory obligations. The shortfall, $\widetilde{R}^{(m)}(t_i)$, is recapitalized from shareholders.

- $\widetilde{A}^{(m)}(t_i) - \widetilde{V}^{(m)}(t_i) \geq 0$: no default and enough capital to meet regulatory requirements. The excess capital is distributed to shareholders as a dividend, $\widetilde{D}^{(m)}(t_i)$.

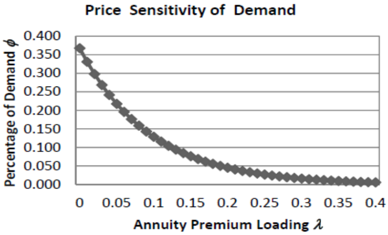
Annuity Demand

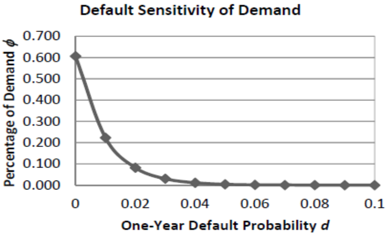
- Exponential demand function (Zimmer *et al.*, 2009, 2011)
  - Default sensitivity α, price sensitivity β
  - Annuity premium loading factor γ^P
  - Cumulative default probability d

$$\phi^*(\gamma^P, d) = e^{(\alpha \cdot d + \beta \cdot \gamma^P + \theta)}.$$

- The number n_0 of annuities sold at time-0
  - n_m is the total market size

$$n_0 = n_m \cdot \phi^*(\gamma^P, d).$$





Economic Balance Sheet (EBS)

- Frictional Costs: $\widetilde{FC}^{(m)}(t) = \rho \cdot [\widetilde{V}^{(m)}(t) - \widetilde{V}_s^{(m)}(t)]$
- Recapitalization Costs: $\widetilde{FC}_R^{(m)}(t) = \psi \cdot \widetilde{R}^{(m)}(t)$
- LLPO: see slide 16
- $X(0)$ represents shareholder value at time-0

Assets	Liabilities
Π	$V_s^{(m)}(0)$
	$\widetilde{PV}_{FC}^{(m)}(0)$
	$\widetilde{PV}_{FC^R}^{(m)}(0)$
	$\widetilde{PV}_{E}^{(m)}(0)$
	$-LLPO(0)$
	$X(0)$

Market-Consistent Embedded Value (MCEV)

- The present value of future profits

$$\widetilde{FP}^{(m)}(t_i) = \sum_{t_s=t_{i+1}}^{t_{n-1}} \left[\left(\widetilde{V}^{(m)}(t_s) - \widetilde{V}^{(m)}(t_{s-1}) \right) + i \cdot \widetilde{A}^{(m)}(t_{s-1}) + \widetilde{CF}^{(m)}(t_s) \right] \cdot \nu(t_i, t_s).$$

- The Value of the In-Force business (VIF)

$$VIF(t) = \widetilde{FP}^{(m)}(t_i) - \widetilde{PV}^{(m)}_{FC}(t_i) - \widetilde{PV}^{(m)}_{FC^R}(t_i) + \widetilde{LLPO}(t_i).$$

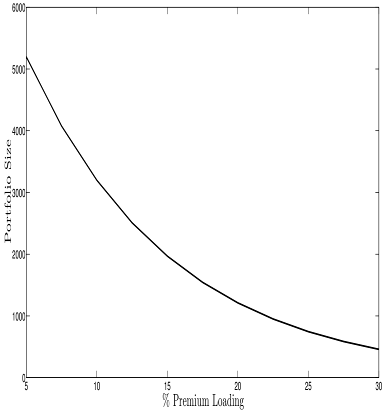
- MCEV at time-t_i is

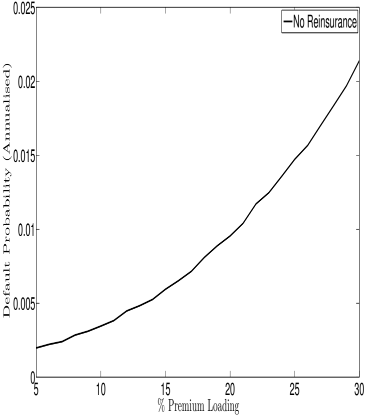
$$MCEV(t_i) = VIF(t_i) + E^Q(t_i), \qquad (2)$$

- where $E^Q(t_i)$ is the time-t_i equity of the insurer.
- Valuation at $t = 0$: $E^Q(t_i) = 0$, SHV is $VIF(t_i)$.

Results

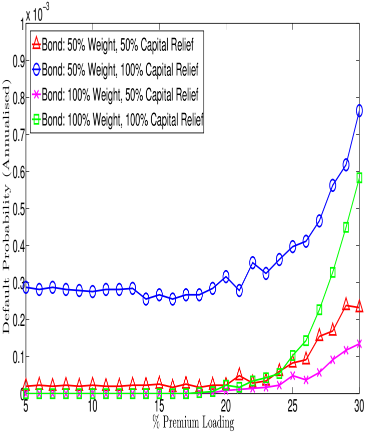
- Assume fixed one-year default probability of 0.5%.
- Portfolio size depends on premium loading.
- Insurer's actual default probability depends on premium loading.



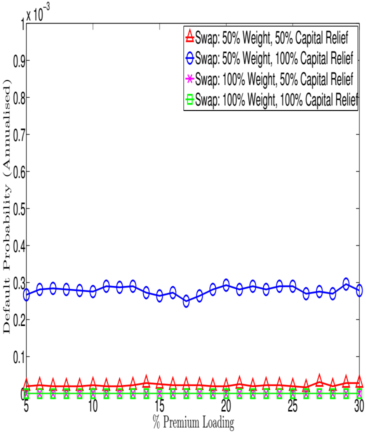


Results: One-Year Default Probabilities

- Bond: higher premium loading - smaller portfolio size - higher default prob.
- Swap: not a problem, idiosyncratic risk is hedged
- No effect of capital relief when insurer is fully hedged.



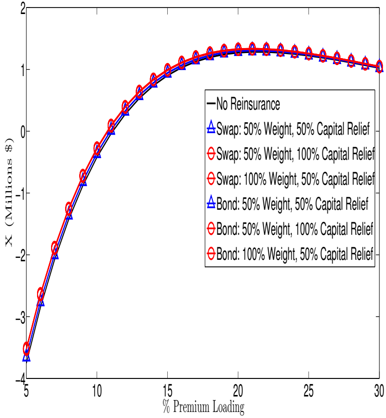
(c) Bond



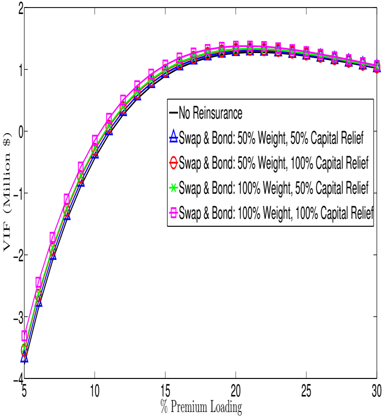
(d) Swap

Results: Shareholder Value

- Longevity risk transfer: small gains to the expected VIF and EV values for any fixed premium loading.
- Reason: reduction of frictional costs.



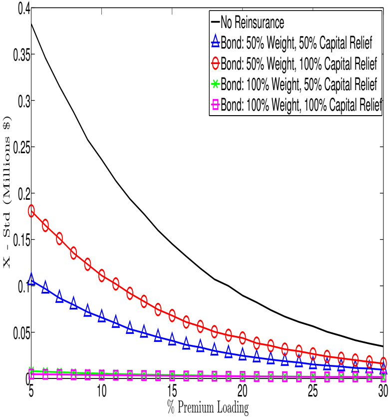
(e) Economic Balance Sheet



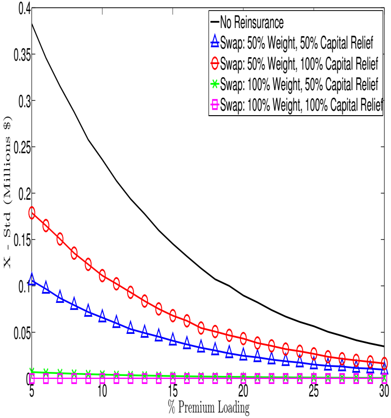
(f) MCEV

Results: Volatility of Shareholder Value

- Longevity risk transfer reduces the volatility of SHV
- Here: SHV from the **Economic Balance Sheet**



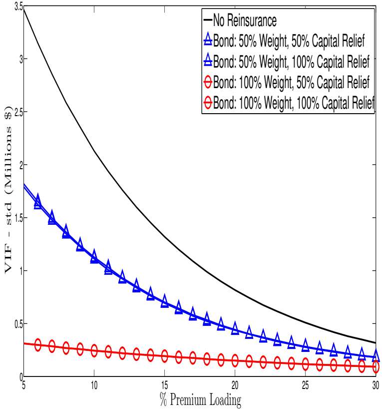
(g) X (Std) Bond



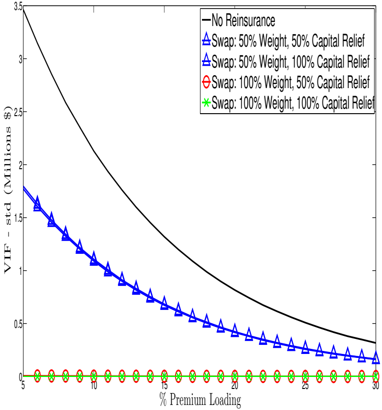
(h) X (Std) Swap

Results: Volatility of Shareholder Value

- Longevity risk transfer reduces the volatility of SHV
- Here: **MCEV** (=VIF in our model)



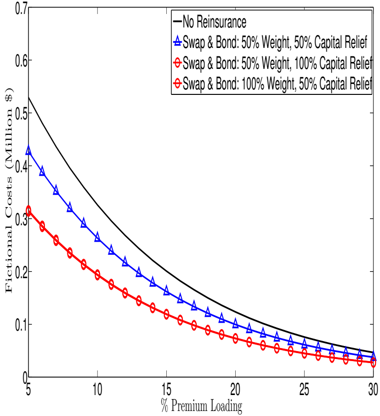
(i) MCEV (Std) Bond



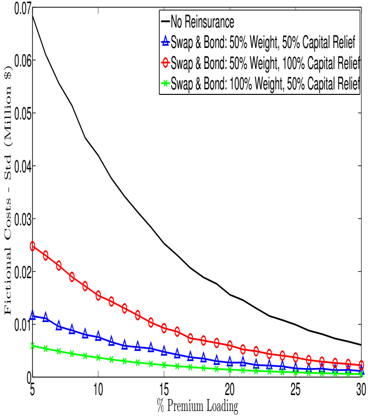
(j) MCEV (Std) Swap

Results: Frictional Costs

- FC: $\widetilde{FC}^{(m)}(t) = \rho \cdot [\widetilde{V}^{(m)}(t) - \widetilde{V}_s^{(m)}(t)]$
- Longevity risk transfer reduces the expected value and the volatility of frictional costs.



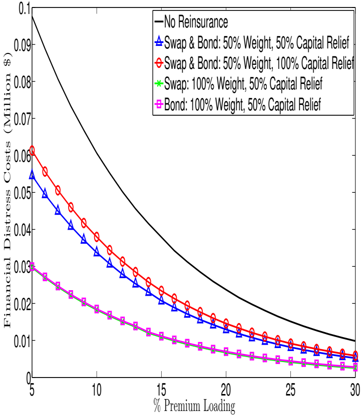
(k) Frictional Costs



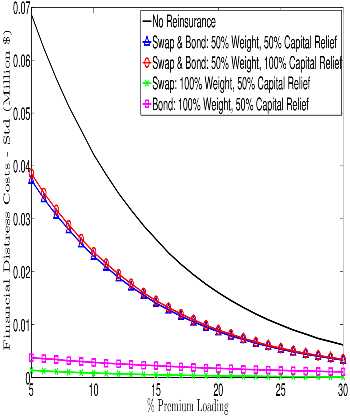
(l) Frictional Costs (std)

Results: Financial Distress Costs

- Longevity risk transfer reduces the expected value and the volatility of recapitalization costs.



(m) Financial Distress Costs



(n) Financial Distress Costs (std)

Conclusion

- Stochastic mortality model with systematic and idiosyncratic longevity risk
- Test risk transfer strategies
  - Survivor Swap : indemnity based
  - Survivor Bond : index based
- EBS and MCEV valuation methods
- Maintain Solvency II SCR
- Benefits of Longevity risk transfer
  - Reduce the insurer's default probability.
  - Increases shareholder value and,
  - Reduces the volatility of the shareholder value.
  - Reduces the friction costs and the volatility of friction costs.
  - Reduces the volatility of dividend payment and recapitalization requirements.

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