ROBUST HEDGING OF LONGEVITY RISK

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Plan

- Intro + model
- Recalibration risk – introduction
- Robustness questions – index hedging
- Are some hedging instruments more robust than others?
- Static Delta and Nuga hedging
- Discussion
Background

- Annuity providers and pension plans
- Exposure to longevity risk
  - systematic risk (underlying mortality rates)
  - binomial risk (lives)
  - concentration risk (amounts)
- Alongside: interest rate risk, equity risk ....
Hedging problem 1

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- \( S(t, 65) \) = proportion still alive at \( t \)
- \( PV = \sum_{t=1}^{\infty} e^{-rt} S(t, 65) \)
- Objective: Hedge longevity risk in \( PV \)
Hedging problem 2

Annuity provider seeks to hedge its exposure to longevity risk

- Large cohort aged 65 at time 0
- Equal, level annuities payable for life
- \( S(t, 65) = \) proportion still alive at \( t \)
- Deficit \( D(t) = MCV_{Liabs}(t) - MCV_{Assets}(t) \)
- Objective: Hedge longevity risk in \( D(T) \)
  e.g. \( T = 1 \) under Solvency II
Hedging problem 3

Pension plan

- Cohort now aged 55
- Plan will buy annuities at age 65
- Objective: hedge the longevity risk in the annuity price
Options for hedging

- Customised hedges:
  - e.g. longevity swap
  - floating leg linked to OWN cashflows
  - indemnification

- Index-based hedges:
  - Standardised contracts
  - e.g. Linked to a national index
Focus of this talk

Index-based hedges

- Customised longevity swaps only available to very large pension plans
- Index-based hedges
  - smaller schemes
  - better value for money for large plans ???
- Quantity of hedging instrument
  Hedge effectiveness
  Price

How confident are we in these quantities? ⇒ ROBUSTNESS
Simple example

- Static value hedge: $t = 0 \rightarrow T$
- $a_k(T, x) = \text{population } k \text{ annuity value at } T$
- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred longevity swap

$$H(T) = a_k(T, x) - \hat{a}_k^{\text{fxd}}(0, T, x)$$

$$\hat{a}_k^{\text{fxd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$$

- $k = 2 \text{ (CMI)} \Rightarrow \text{CUSTOMISED hedge}$
- $k = 1 \text{ (E&W)} \Rightarrow \text{INDEX hedge}$
Hedging: basic idea

- $L = \text{liability value}$
- $H = \text{value of hedging instrument}$

Objective: minimise $\text{Var}(\text{deficit}) = \text{Var}(L + hH)$

$\Rightarrow$ hedge ratio, $h = -\frac{\text{Cov}(L, H)}{\text{Var}(H)} = -\rho \frac{\text{S.D.}(L)}{\text{S.D.}(H)}$

Hedge effectiveness $= 1 - \frac{\text{Var}(L + hH)}{\text{Var}(L)} = \rho^2$

More general: multiple assets

$\Rightarrow$ minimise $\text{Var}(L + h_1H_1 + \ldots + h_nH_n)$
Simple example: APC model (Cairns et al., 2011a)

\[ m_k(t, x) = \text{population } k \text{ death rate} \]

\[ \log m_k(t, x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t - x) \]

\( \beta^{(1)}(x), \beta^{(2)}(x) \) population 1 and 2 age effects

\( \kappa^{(1)}(t), \kappa^{(2)}(t) \) period effects; mean reverting spread

\( \gamma^{(1)}(c), \gamma^{(2)}(c) \) cohort effects

Key: \( \nu_k = \kappa^{(1)}(t), \kappa^{(2)}(t) \) long term trend
Realism: valuation model $\neq$ simulation model

- (Re-)calibration using data up to $T \Rightarrow$ realistic!
- Valuers just observe historical mortality plus one future sample path of mortality from 0 to $T$
  \[ \Rightarrow \text{do not know the “true” simulation/true model} \]
- Using true model $\Rightarrow$ too optimistic (??)  
  c.f. Black-Scholes
- Valuation model $+$ calibration window $\Rightarrow$ Knightian Uncertainty
Recalibration risk – example (random walk)

- You will recalibrate at $T$
- Recalibration depends on as yet unknown experience from 0 to $T$
- Recalibration depends on length of lookback window
Hedge Effectiveness: (Cairns et al., 2011b; Longevity 6)

Key conclusions: index-based hedging

- Recalibration $\Rightarrow$ risk $\uparrow$

- BUT hedge effectiveness also $\uparrow$

**WHY?**

Additional trend risk is common to both populations.

$$a_k(T, x) \approx f(\beta^{(k)}_x, \kappa^{(k)}_T, \gamma^{(k)}_{T-x+1}, \nu_\kappa)$$
Robustness

How robust are estimates of:

- Optimal hedge ratios $h_1, \ldots, h_n$
- Hedge effectiveness
- Initial hedge instrument prices $\pi(H_1), \ldots, \pi(H_n)$

... relative to ...
Robustness

How robust are key quantities relative to

- Treatment of parameter risk
- Treatment of population basis risk
- Valuation model: recalibration risk
- Poisson risk
- Use of latest EW data
- Simulation model + calibration
Modelling Variants

- PC: Full parameter certainty (PC);
  Valuation Model NOT recalibrated in 2015

- PC-R: As full PC
  Except: Valuation Model recalibrated in 2015

- PU: Full parameter uncertainty with recalibration

- PU-Poi: Full PU with recalibration + Poisson risk
Hedging options

- Recall: Liability, $L = a_2(T, 65)$ (CMI)

- Hedging instrument (ref England & Wales):
  - $H = a_1(T, x) - a_1^{\text{fxd}}(0, T, x)$
    
    OR

  - $q$-Forward maturing at $T$ (www.LLMA.com)
    
    $H = q_1(T, x) - q_1^F(0, T, x)$

- Both cases: for a range of reference ages $x$
Robustness of Hedge Ratios

PC $\rightarrow$ PC-R not robust; PC-R $\rightarrow$ PU robust
deferred longevity swaps better than maturing $q$-Forwards
Robustness relative to recalibration window, $W$

Maturing $q$-forwards

Deferred Longevity Swaps

Deferred longevity swaps better than maturing $q$-forwards
Robustness relative to recalibration window, $W$

Longevity swaps are more robust:

- Liability, $L$, and longevity swap, $H$, depend on
  - $\kappa^{(1)}_T$ and $\nu_\kappa$
  - BUT in differing proportions $\Rightarrow$ single $H$ not robust

- Maturing $q$-Forward depends on $\kappa^{(1)}_T$ only
  $\Rightarrow$ even less robust

- Possible market solution:
  $$(0, T + U, x) \quad q$-Forward, cash settled at $T$$
Robustness relative to recalibration window, $W$

Hedging with Cash-Settled, Long-Maturity $q$-Forwards

$T + U$ $q$-Forward is cash settled at time $T$

$\Rightarrow$ value depends on $\kappa_t^{(1)}$ and $\nu_\kappa$
Robustness relative to recalibration window, $W$

- If we know $W$, then $\nu_\kappa$ linear in $\kappa_T^{(1)}$
  $\Rightarrow$ one hedging instrument sufficient

- If $W$ is not known
  or, $\nu_\kappa$ determined by other methods
  $\Rightarrow$ two hedging instruments are required
  $\Rightarrow$ Delta and "Nuga" hedging
Delta and Nuga Hedging

Recall: \( a_k(T, x) \approx f(\beta_{[x]}^{(k)}, \kappa_T^{(k)}, \gamma_{T-x+1}^{(k)}, \nu_{\kappa}) \)

Liability: \( L = a_2(T, x) \).

Hedge instruments:

\[
H_1 = q_1(T, x_1) - q_1^{\text{fxd}}(0, T, x_1) \quad \to h_1 \text{ units}
\]
\[
H_2 = q_1(T + U, x_2) - q_1^{\text{fxd}}(0, T + U, x_2) \quad \to h_2 \text{ units}
\]

\( (H_2 \text{ cash settled at } T) \)
Delta and Nuga hedging ⇒ require

Deltas: \[ \alpha \frac{\partial L}{\partial \kappa^{(2)}} = -h_1 \frac{\partial H_1}{\partial \kappa^{(1)}} - h_2 \frac{\partial H_2}{\partial \kappa^{(1)}} \]

and Nugas: \[ \frac{\partial L}{\partial \nu_\kappa} = -h_1 \frac{\partial H_1}{\partial \nu_\kappa} - h_2 \frac{\partial H_2}{\partial \nu_\kappa} \]

where \( \alpha = \text{Cov}(\kappa_T^{(1)}, \kappa_T^{(2)}) / \text{Var}(\kappa_T^{(1)}) \).

Concept:

same idea as Vega hedging in equity derivatives

– hedging against changes in a parameter that is supposed to be constant.
Numerical example: \( L = a_2(T, 65), T = 10 \)

Four strategies:

A: No hedging

B: \( H_1 \) only; \( h_1 \) optimal for \( W = 20 \)

C: \( H_1 \) only; \( h_1 \) optimal for \( W = 35 \)

D: \( H_1 \) and \( H_2 \); Delta and Nuga hedging
Numerical example: \( L = a_2(T, 65), T = 10 \)

\[
q\cdot F(T, 64) \quad q\cdot F(T + T, 74)
\]

<table>
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<th>Strategy</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( Var(\text{Deficit}) )</th>
<th>Hedge Eff.</th>
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Numerical example: discussion

- Nonlinearities $\Rightarrow D < B$ instead of $D = B$

- BUT
  
  - $W = 20 \Rightarrow$
    
    $D$ is nearly optimal
    
    $C$ is much worse
  
  - $W = 35 \Rightarrow$
    
    $D$ is nearly optimal
    
    $B$ is much worse
Robustness relative to other factors

Results are robust relative to:

- inclusion of parameter uncertainty in $\beta_{x}^{(k)}$, $\kappa_{t}^{(k)}$, $\gamma_{c}^{(k)}$
- pension plan’s own small-population Poisson risk
- index population: EW-size Poisson risk, maybe smaller
- CMI data up to 2005 + EW data up to 2005
  versus
  CMI data up to 2005 + EW data up to 2008
Conclusions

Robust hedging requires *inclusion* of

- Recalibration risk (Nuga)
- Careful treatment of recalibration window
- Long-dated hedging instruments to handle *Nuga* risk

Results appear to be robust relative to

- Poisson risk
- Parameter uncertainty (other than recalibration risk)
- Treatment of latest data

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