Modelling and Evaluating Longevity Risk
A Case Study UK

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LUH-Kolloquium "Versicherungs- und Finanzmathematik" - Mortality and Longevity
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Risks associated with Longevity

- Mortality risk
  - Misestimation/error risk
  - Volatility risk
  - Trend risk

- Dependant's pension
  - Who is a dependant?
  - Age difference
  - Proportion married
  - Dependant's benefit

- Investment risk
  - Asset risk
  - Inflation risk

- Regulatory alterations (e.g. unisex, tax, Solvency II, …)
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Drivers of Mortality
...apart from gender and age

- Health
- Country, Postcode
- Occupation
- Socioeconomy
- Marital Status
- Genes
- External drivers
- ...
Health

Survival by Disease at Age 65 - Female, UK in %

Significant differences in shape of the survival curves
Remaining Period Life Expectancy at Age 65 in 2006 by Country

<table>
<thead>
<tr>
<th>Country</th>
<th>Period Life Expectancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poland</td>
<td>12</td>
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<tr>
<td>Sweden</td>
<td>13</td>
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<tr>
<td>Denmark</td>
<td>14</td>
</tr>
<tr>
<td>Germany</td>
<td>15</td>
</tr>
<tr>
<td>Italy</td>
<td>16</td>
</tr>
<tr>
<td>France</td>
<td>17</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>18</td>
</tr>
<tr>
<td>Switzerland</td>
<td>19</td>
</tr>
</tbody>
</table>

**Period life expectancy** is the average number of years a person would live, if he or she experienced the mortality rates valid for a certain time period throughout his or her life. It makes no allowance for any later actual or projected changes in mortality.

Source: WHO - European health for all database (HFA-DB), http://data.euro.who.int/hfadb/
Remaining Period Life Expectancy at Age 65 in 2006 by Country

Country Differences in life expectancy by country

Period life expectancy is the average number of years a person would live, if he or she experienced the mortality rates valid for a certain time period throughout his or her life. It makes no allowance for any later actual or projected changes in mortality.

Source: WHO - European health for all database (HFA-DB), http://data.euro.who.int/hfadb/
Country
Systematic differences in mortality by country

SDR(Agegroup 60 – 74, Males) by Country and Year - Circulatory Diseases

Source: WHO - European health for all database (HFA-DB), http://data.euro.who.int/hfadb/

Standardized Death Rate (SDR): The number of deaths per 100,000 people of a specified population during 1 year
Socioeconomy
Systematic differences in mortality by socioeconomy

Remaining Period Life Expectancy at Age 60 – Male, UK

Source: Office for National Statistics – Longitudinal Study
Risks associated with Longevity

► Mortality risk
  • Misestimation/error risk
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  • Who is a dependant?
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► Investment risk
  • Asset risk
  • Inflation risk

► Regulatory alterations (e.g. unisex, tax, Solvency II,…)
Rapid Mortality Improvements over the 20th Century
United Kingdom

<table>
<thead>
<tr>
<th>Year</th>
<th>Period life expectancy at birth - Males</th>
<th>Period life expectancy at birth - Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>1901</td>
<td>46 *</td>
<td>50 *</td>
</tr>
<tr>
<td>2009</td>
<td>78 **</td>
<td>82 **</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>$q_0$ - Males</th>
<th>$65 P_0$ - Males</th>
<th>$q_0$ - Females</th>
<th>$65 P_0$ - Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>1901</td>
<td>16% *</td>
<td>36% ***</td>
<td>13% *</td>
<td>43% ***</td>
</tr>
<tr>
<td>2009</td>
<td>&lt; 0.5%***</td>
<td>85% ***</td>
<td>&lt; 0.5%***</td>
<td>90% ***</td>
</tr>
</tbody>
</table>

Source: Office for National Statistics
* The health of adult Britain 1841-1994 (1997a)
** Statistical Bulletin: Older People’s Day 2011
*** UK Interim Life Tables 1980-82 to 2008-10 (2011)

Today, most of us expect to reach their retirement age!
### Rapid Mortality Improvements over the 20th Century

#### England and Wales

<table>
<thead>
<tr>
<th>Age group</th>
<th>Males</th>
<th>Females</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>82%</td>
<td>86%</td>
<td>75%</td>
<td>74%</td>
</tr>
<tr>
<td>21-40</td>
<td>74%</td>
<td>80%</td>
<td>31%</td>
<td>47%</td>
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<tr>
<td>41-60</td>
<td>44%</td>
<td>60%</td>
<td>56%</td>
<td>49%</td>
</tr>
<tr>
<td>61-80</td>
<td>15%</td>
<td>36%</td>
<td>57%</td>
<td>54%</td>
</tr>
</tbody>
</table>


Shift of high improvement rates of mortality to higher ages.
Methods of Mortality Projection
Government Actuary's Department Review 2001

Process-based methods
Concentrate on the factors that determine deaths and attempt to model mortality rates from a bio-medical perspective
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Concentrate on the factors that determine deaths and attempt to model mortality rates from a bio-medical perspective

Explanatory-based methods
Employ a causal forecasting approach, e.g. using econometric techniques based on variables such as (socio)economic or environmental factors
Methods of Mortality Projection
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Process-based methods
Concentrate on the factors that determine deaths and attempt to model mortality rates from a bio-medical perspective.

Explanatory-based methods
Employ a causal forecasting approach, e.g. using econometric techniques based on variables such as (socio)economic or environmental factors.

Extrapolative methods
- Project historical trends in mortality into the future
- Parametric models
- Targeting models
- Trend models
- All these models can either be deterministic or stochastic
A Journey through the History of UK Improvements
... starting in 1980

Historic Improvements

- CMI Report 10 (1990): 80 series of base tables + Allowance for improvement by reduction factor

- CMI Report 17 (1999): 92 series of base tables + Updated model for allowance for improvement by reduction factor

- The cohort effect


- CMI Mortality Projections Model (2009)
The 80 series of mortality tables

- Data: UK and Republic of Ireland, 1979-1982, insurance data

- CMI Report 10: Complete tables of values of qx for male and female valid in 1980

- Allowance for improving mortality \( RF(x, t) = \frac{q(x, t)}{q(x, 0)}, t \geq 0 \geq 1980 \)
  - Reduction factor on base tables valid in 1980
  - Idea: \( RF(x, t) = \alpha(x) + (1 - \alpha(x)) \exp(-\beta t) \)
    - Limiting value
    - Lower bound
    - Determines speed of convergence to limiting value

where for all \( x \geq 0: 0 < \alpha(x) \leq 1 \) and \( \beta > 0 \)
The 80 series of mortality tables

- Data: UK and Republic of Ireland, 1979-1982, insurance data

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- Allowance for improving mortality

\[ RF(x,t) = \frac{q(x,t)}{q(x,0)}, \quad t \geq 0 \approx 1980 \]

- Reduction factor on base tables valid in 1980

\[ \alpha(x) \]
The 92 series of mortality tables
Allowance for improvement

- Data: UK and Republic of Ireland, 1991-1994, insurance data

- CMI Report 10: Complete tables of values of qx for male and female valid in 1992

- Allowance for improving mortality – Updated model
Comparison
RF80 versus RF92

Period Life Expectancy at Age 65: Male, UK

in Years


18
17
16
15
14
13
12
11
10

RF80  RF92  Historic data
The Cohort Effect

Historic Annual Rates of Improvement: Male, UK

- "Cohort Effect":
  - People born between 1925 and 1945 have benefited from faster mortality improvements than those born in adjacent generations
  - Centered on the generation born about 1931

- Reasons: "The healthy generation"
  - Not involved in active service in World War II
  - Better childhood health
  - Smoked fewer cigarettes than the previous generation
  - Benefited from the introduction of the state education system and the National Health service
  - Have tended to have fewer children than the previous generation
The Cohort Effect

“Mortality in the Next Millennium”, Richards Willets (12/1999)

Cohort Effect:
- People born between 1925 and 1945 have benefited from faster mortality improvements than those born in adjacent generations.
- Centered on the generation born about 1931.
  - “The healthy generation”
    - Not involved in active service in World War II
    - Better childhood health
    - Smoked fewer cigarettes than the previous generation
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    - Have tended to have fewer children than the previous generation

Historic Annual Rates of Improvement: Male, UK in %

--- | --- | --- | --- | --- | --- | --- | --- | ---
Age  | 20  | 28  | 36  | 44  | 52  | 60  | 68  | 76  | 84  | 92  | 100

- 5.00%-6.00%
- 4.00%-5.00%
- 3.00%-4.00%
- 2.00%-3.00%
- 1.00%-2.00%
- 0.00%-1.00%
- -1.00%-0.00%
- -2.00%--1.00%
- -3.00%--2.00%
- -4.00%--3.00%
- -5.00%--4.00%
CMI Working Paper 1:
- Projections incorporating the cohort effect (sc, mc, lc)
- Interim basis for adjusting the 92 series of mortality tables for cohort effects

Cohort period
- Period over which the cohort effect for the 1926 cohort is assumed to persist
  - Short Cohort Period: 1926 – 2010
  - Medium Cohort Period: 1926 - 2020
  - Long Cohort Period: 1926 – 2040
- From 2001, the improvement rates reduce linearly to the end of the cohort period where they equal those derived from the original “92” Series projections.

Width of the cohort effect:
- Between 1992 and 2000: cohort includes births between 1910 and 1942
- After 2000: the ‘width’ of the cohort effect is reduced so that by the end of the cohort period it includes only one year, which relates to lives born in 1926.
Cohort Improvements
An interim basis for adjusting the 92 series for cohort effects

Annual Rates of Improvement: sc Projections in %
Cohort Improvements
An interim basis for adjusting the 92 series for cohort effects

Annual Rates of Improvement: mc Projections in %
Cohort Improvements
An interim basis for adjusting the 92 series for cohort effects

Annual Rates of Improvement: lc Projections

<table>
<thead>
<tr>
<th>Year</th>
<th>20</th>
<th>28</th>
<th>36</th>
<th>44</th>
<th>52</th>
<th>60</th>
<th>68</th>
<th>76</th>
<th>84</th>
<th>92</th>
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<td>2060</td>
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<td>2068</td>
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</tbody>
</table>
Comparison
80, 92 and cohort improvements

Period Life Expectancy at Age 65: Male, UK

in Years


RF80 RF92 Historic data sc mc lc

Modelling and Evaluating Longevity Risk
CMI Mortality Projections Model (2009)

Motivation

- Short & Medium Cohort now imply rapid tail-off in rates of improvement in future mortality

- Out of date as it was based on data up to 1999

- The goal of the Working Party was to produce a model which
  
  • Shares desirable features of the Interim Cohort Projections
  • Reflects the latest experience on trends in mortality
  • Is relatively straightforward to understand and describe
  • Allows users to flexibility to modify projections to suit their own views and purpose
  • Can be regularly updated over time to reflect emerging experience
Historic annual rate of mortality improvement

\[ i(t, x), x = \text{age}, t \leq 0 (\text{time}, 0 = \text{today or Baseyear of projection}) \]

Smoothing of historic rates using a P-spline model

Split of the smoothed rates into three components: an Age-, a Period- and a Cohort Component using a B-spline model

\[ i(t, x) = a(x) + b(t) + c(t - x) \]

Separate projection of Age/Period Component and Cohort Component

Basic concept: The **Initial Rates** (recently observed experience) blend over into a **Long Term Rate** of mortality improvements
i(0, x) = IR(x)

\[ i\left(\frac{T(x)}{2}, x\right) = IR(x)P + LTR(x)(1 - P) \]

i(T(x), x) = LTR(x)

General

\[ i(t, x) = IR(x)a(t) + LTR(x)(1 - a(t)) \]

\[ a(t) = \begin{cases} 
\frac{x_1}{PoC}t^3 + \frac{x_2}{PoC}t^2 + \frac{x_3}{PoC}t + x_4, & t \leq T(x) \\
0, & t > T(x) 
\end{cases} \]

a'(T(x)) = 0
CMI Mortality Projections Model (2009)
Core Level

Only two parameters have to be specified in the Core Level

- **Long Term Rate** for the Age/Period Component
  - More subjective
  - No standard assumption
  - Very high impact

- **Constant Addition**
  - Applied to all ages and calendar years of the projection
  - For incorporating a degree of prudence or for valuation purposes
  - Often set to 0

Long Term Rate for Cohort Component set to 0.
When running the tool in the Advanced Level, the Long Term Rate is depending on age and cohort.

In addition, you have to specify:
- Initial Rates of mortality improvement
- Convergence parameters
  - Proportion of Change Remaining at Midpoint
  - Period of Convergence
- Constant Addition

All parameters may be specified for every single age and every cohort separately (exception: Constant Addition).

Technically speaking, there are more than 1,000 parameters for both males and females.
Structuring the shape of convergence
Constant Addition vs LTR

Annual improvement at Age 60: Male, UK in %

Higher LTR + Constant Addition

Higher LTR

0.00% 0.50% 1.00% 1.50% 2.00% 2.50% 3.00% 3.50% 4.00%

For a negative initial rate a longer Period of Convergence does not mean adding conservatism!
For a negative initial rate a longer Period of Convergence does not mean adding conservatism!
Structuring the shape of convergence
Proportions of Convergence Remaining at Midpoint

For a negative initial rate a higher proportion does not mean adding conservatism!
Projections
CMI tool and old cohort projections

More adequate in general
All cohorts are projected
Thank you for your attention!