HEDGING SYSTEMATIC MORTALITY RISK WITH MORTALITY DERIVATIVES

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## OUTLINE

| Introduction | • Mortality risk management  
<table>
<thead>
<tr>
<th></th>
<th>• Valuation</th>
</tr>
</thead>
</table>
| Mortality risk | • Unsystematic risk  
|             | • Systematic risk |
| Hedging mortality risk | • Survivors swaps  
|             | • A modeling framework |
| Illiquid assets | • Risk-minimization  
|             | • Inherent risk and survivor swaps |
RISK MANAGEMENT IN LIFE INSURANCE

Current situation: Market based valuation of assets and liabilities

Financial risks: Partly controlled via investments, derivatives etc

Mortality and longevity risk:
An essential non-hedged and non-hedgeable (?) risk for pension funds

Solvency II: Capital requirements derived from properties for actual products, inherent risks and investments

Capital requirements and risk margins for life annuities is to be calculated using a 20 percent reduction in mortality rates

Mortality derivatives could lead to lower capital requirements!
(e.g. survivor swaps, survivor bonds, other constructions)
VALUATION AND MORTALITY RISK MANAGEMENT

Key issues

- Expected future mortality development (trend, volatility)
- Level of risk premium
- Natural buyers and sellers of mortality risk

Illustration of importance of trend for valuation

Survival probability (initial age 65)

Example with "old" mortality:

<table>
<thead>
<tr>
<th>Trend</th>
<th>$V$</th>
<th>$E$</th>
<th>$25p_{65}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>10.66</td>
<td>80.3</td>
<td>12.5%</td>
</tr>
<tr>
<td>1%</td>
<td>11.04</td>
<td>81.2</td>
<td>17.3%</td>
</tr>
<tr>
<td>2%</td>
<td>11.46</td>
<td>82.3</td>
<td>22.7%</td>
</tr>
<tr>
<td>3%</td>
<td>11.92</td>
<td>83.6</td>
<td>28.4%</td>
</tr>
<tr>
<td>4%</td>
<td>12.43</td>
<td>85.1</td>
<td>34.3%</td>
</tr>
</tbody>
</table>
IMPROVEMENT RATES FOR DIFFERENT PERIODS

Observed yearly relative decline, Denmark

Yearly average improvement rates depend on the period considered. Largest improvement rates observed in the period 1990-2006

Danish FSA has introduced a mortality and longevity benchmark

Benchmark for current mortality estimated from data for insured individuals with 5 years of data

Pension funds perform yearly statistical tests whether they derive from benchmark

Benchmark for future trend estimated from total Danish population (20 years of data)
BENCHMARK - OE-RATES 2006-2009 - MALES

- Benchmark determined using splines from (log) linear regression
- Special model for high age mortality - logistic model
- "Kannisto model" suggested by Thatcher et al, 1998
- Not exponential increase of mortality after age 100
APPROACH TO LONGEVITY MODELING IN DENMARK

Benchmark
- Future trend assumption based on observations for 20 years
- Age-dependent yearly decline
- Different decline rates for males and females

- Trend and current mortality lead to **best estimate**

**Solvency 2: 20 % reduction of mortality rates**

<table>
<thead>
<tr>
<th>Age</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>84,0</td>
<td>86,5</td>
</tr>
<tr>
<td>50</td>
<td>83,5</td>
<td>86,2</td>
</tr>
<tr>
<td>60</td>
<td>83,8</td>
<td>86,6</td>
</tr>
<tr>
<td>65</td>
<td>84,3</td>
<td>87,0</td>
</tr>
<tr>
<td>70</td>
<td>85,1</td>
<td>87,7</td>
</tr>
<tr>
<td>80</td>
<td>88,1</td>
<td>90,0</td>
</tr>
<tr>
<td>90</td>
<td>93,9</td>
<td>94,7</td>
</tr>
</tbody>
</table>
TWO TYPES OF MORTALITY RISK
TWO FUNDAMENTALLY DIFFERENT TYPES OF MORTALITY RISK

Systematic mortality risk:
- Unexpected changes in underlying mortality intensities and expected life times:
  - Same effect on all individuals
  - NOT diversifiable
- Long-term risk
- Risk management solutions: new products, mortality derivatives

Unsystematic mortality risk:
- Randomness of deaths given underlying intensities:
  - Law of large numbers (Jakob Bernoulli, 1654-1705)
  - Risk is diversifiable
- Short and long term risk

Interaction is not trivial!
LONG TERM SIMULATION OF NUMBER OF SURVIVORS

Simulated number of survivors at age 85, given initial age 30

<table>
<thead>
<tr>
<th>100 individuals</th>
<th>1000 individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected number of survivors: 37</td>
<td>Expected number of survivors: 374</td>
</tr>
<tr>
<td>Std. dev. without systematic risk: 4.9</td>
<td>Std. dev. without systematic risk: 15.4</td>
</tr>
<tr>
<td>Std. dev. with systematic risk: 7.3</td>
<td>Std. dev. with systematic risk: 57.7</td>
</tr>
</tbody>
</table>
SHORT TERM SIMULATION OF NUMBER OF SURVIVORS
- PORTFOLIO OF RETIRED

Simulated number of survivors at age 85, given initial age 75

<table>
<thead>
<tr>
<th></th>
<th>100 individuals</th>
<th>1000 individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected number of survivors:</td>
<td>43.5</td>
<td>435</td>
</tr>
<tr>
<td>Std. dev. without systematic risk:</td>
<td>4.9</td>
<td>15.8</td>
</tr>
<tr>
<td>Std. dev. with systematic risk:</td>
<td>5.2</td>
<td>21.5</td>
</tr>
</tbody>
</table>
SURVIVOR SWAPS AND MORTALITY RISK MODELING
SURVIVOR SWAPS

Fix a portfolio of (insured) lives:

- Actual number of survivors (variable leg)
- Expected number of survivors (fixed leg)
- Pension fund receives difference, if > 0
- Pension fund pays difference, if < 0
- Possible expiry T

"Expected" number may include a risk premium
SURVIVOR SWAP – UNDERLYING PORTFOLIO

Possibilities

<table>
<thead>
<tr>
<th>Own portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other “small” portfolio</td>
</tr>
<tr>
<td>Large reference population</td>
</tr>
</tbody>
</table>

Choice of underlying portfolio:

- Survivor swap linked to own portfolio may provide “perfect hedge” but may be less liquid *(no market)*

- Using a “small” portfolio on other lives will typically not provide a good hedge. Unsystematic risk will dominate *(investment instrument)*

- Large reference population gives desired properties if sufficiently long time horizon *(more liquid, possible market?)*
SURVIVOR BONDS

Y (t,x): Number of persons alive at t, aged x at time 0

**Survivor index:** \( S(t,x) = \frac{Y(t,x)}{Y(0,x)} \)

(observable, stochastic process, non-traded)

**Payment process:** \( dB(t) = S(t,x) \, dt, \, t < T \)

*Cannot be replicated with existing instruments!*

Expected relative number of survivors: \( E_Q[S(t,x)] = t p_Q^x \) (choice of Q?)

Price:

\[
E^Q \left[ \int_t^T e^{-\int_t^u r(s)\,ds} S(u,x)\,du \bigg| F(t) \right] = \int_t^T P(t,u) E^Q \left[ S(u,x) \big| F(t) \right] du
\]

**Discounting factor - interest rate risk**

**Longevity risk**
SURVIVOR BONDS: EXAMPLE OF PAYMENTS

Actual number of survivors compared to expected number of survivors (black line) in two different stochastic scenarios (red and blue lines). Left plot shows number of survivors in the insurance portfolio and right plot shows number of survivors in larger portfolio. (Portfolio sizes 10,000 and 100,000)
**SURVIVOR SWAPS**

**Payment process:** \( dB^{\text{swap}}(t) = ( S(t,x) - t^{Q_x} ) dt \), \( t < T \)

"**Variable** payments" Survival probability chosen at time 0! Pricing:

\[
\int_t^T P(t,u)E^Q[S(u,x)|F(t)] du - \int_t^T P(t,u)u^{Q_x} du
\]

**Valuation of variable payments:**
- Similar to a *life annuity*
- Interest rate dependent
- Value reflects mortality development
- Increases if mortality decreases

**Valuation of fixed payments:**
- Similar to a *certain annuity*
- Interest rate dependent
- Price does not depend on mortality development

**For pension fund:**
Swap part of market-based accounting
Consistent market values for all assets and liabilities
SURVIVOR SWAPS – ACTUARIAL OR FINANCIAL INSTRUMENT?

**Actuarial interpretation – own portfolio**
- Match payments from life annuities
- If more annuitants survive, pension fund receives the difference
- Not to be traded?
- Entered with reinsurance company

**Financial interpretation**
- Market value of future payments has similar sensitivity towards mortality/longevity risk as existing liabilities
- Value of survivor swap increases after a longevity stress (Solvency II) ⇒ **Capital relief**
- Trading possibilities?
MORTALITY MODEL AND PORTFOLIOS

Insurance portfolio

Initial mortality

Development process

Future mortality

Population

\[ \mu_1^0(x + t) \]

\[ \zeta_1(x, t) \]

\[ \mu_2^0(x + t) \]

\[ \zeta_2(x, t) \]

Correlated, time-inhomogeneous CIR

Stochastic and time-dependent

\[ \mu_1(x, t) = \mu_1^0(x + t) \zeta_1(x, t) \]

\[ \mu_2(x, t) = \mu_2^0(x + t) \zeta_2(x, t) \]

Intuitive and flexible model with nice analytical properties
NUMERICAL EXAMPLES

Time-inhomogeneous CIR model known from finance

\[ d\zeta(x,t) = (\gamma(x,t) - \delta(x,t)\zeta(x,t))dt + \sigma(x,t)\sqrt{\zeta(x,t)}dW^\mu(t) \]

Parameterization

<table>
<thead>
<tr>
<th>$\delta(x,t)$</th>
<th>$\gamma(x,t)$</th>
<th>$\sigma(x,t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>$\tilde{\delta}$</td>
<td>$\tilde{\delta} e^{-\tilde{\gamma}t}$</td>
</tr>
<tr>
<td>Case II</td>
<td>$\tilde{\gamma}$</td>
<td>$\frac{1}{2} \tilde{\sigma}^2$</td>
</tr>
</tbody>
</table>

Quantiles, time horizon 20 years:

<table>
<thead>
<tr>
<th>$\tilde{\delta}$</th>
<th>$\tilde{\gamma}$</th>
<th>$\tilde{\sigma}$</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.2</td>
<td>0.008</td>
<td>0.03</td>
<td>0.814</td>
<td>0.856</td>
<td>0.886</td>
<td>0.917</td>
</tr>
<tr>
<td>Case II</td>
<td>0.008</td>
<td>0.02</td>
<td>0.726</td>
<td>0.801</td>
<td>0.854</td>
<td>0.909</td>
<td>0.990</td>
</tr>
</tbody>
</table>
SIMULATION FOR IMPROVEMENT PROCESS (CASE I) (with mean-reversion)
SIMULATION FOR IMPROVEMENT PROCESS (CASE II)

(without mean-reversion)
MIXED DYNAMIC AND STATIC HEDGING
DYNAMIC **RISK-MINIMIZATION** FOR PAYMENT STREAMS

**Idea**  Minimize conditional expected (squared) in- or outflow not generated by \( A \) using a market measure \( Q \)

**Method**  Market value decomposition

\[
V^{*,Q}(t) = V^{*,Q}(0) + \int_{0}^{t} \xi^A(u)dX(u) + \int_{0}^{t} \mathcal{Q}^A(u)dY(u) + L^A(t)
\]

\( \xi^o(t) = \xi^A(t) \)  \( \mathcal{Q}^o(t) = \mathcal{Q}^A(t) \)

\[
\eta^o(t) = V^{*,Q}(t) - A^*(t) - \xi^A(t)X(t) - \mathcal{Q}^A(t)Y(t)
\]

Results are typically intuitive!
A NAIVE MIXED DYNAMIC AND STATIC RISK-MINIMIZING STRATEGY

Set-up with an illiquid asset

$X$ still traded dynamically $Y$ is an illiquid asset traded at fixed times $t_i$ only

A naive approach

\[
\hat{\xi}^o(t) = \xi^o(t) \quad \text{Investment in } X \text{ unchanged}
\]

\[
\hat{\mathcal{G}}^o(t) = \mathcal{G}^o(t_{i-1}) \quad \text{for } t \in (t_{i-1}, t_i) \quad \text{Investment in } Y \text{ fixed at optimal investment at beginning of period}
\]

Does not work if $X$ and $Y$ are correlated or trend in $\mathcal{G}^o(t)$
MIXED DYNAMIC AND STATIC RISK-MINIMIZING STRATEGY

**Trick to handle correlation between X and Y**

Decompose $Y$ with respect to $X$

$$dY(t) = \xi^Y(t)dX(t) + dL^Y(t)$$

**Optimal strategy for** $t \in (t_{i-1}, t_i)$

$$\hat{\xi}^o(t) = \xi^o(t) + \xi^Y(t)(\mathcal{G}^o(t) - \hat{\mathcal{G}}^o(t))$$

$$\hat{\mathcal{G}}^o(t) = E^Q \left[ \int_{t_{i-1}}^{t_i} \mathcal{G}^A(u)dL^Y(u) \Delta L^Y(t_i) \bigg| F(t_{i-1}) \right]$$

$$\hat{\eta}^o(t) = V^{*,Q}(t) - \bar{A}^*(t) - \hat{\xi}^o(t)X(t) - \hat{\mathcal{G}}^o(t)Y(t)$$

Optimal dynamic strategy corrected by hedgeable part of illiquid asset

Risk adjusted average of dynamic strategy on next interval
CASE STUDY WITH SURVIVOR SWAPS
SURVIVOR SWAPS

Survivor swap payment process

\[ dA_{j, t}^{\text{swap}}(x, t) = \left( n_j - N_j(x, t) \right) dt - n_j t \tilde{p}_x dt \]

Illustration

- **Expected number of survivors** (fixed leg)
- **Actual number of survivors** (variable leg)
- **Pension fund receives difference, if > 0**
- **Pension fund pays difference, if < 0**
- **Possible expiry T**
- **Survivor swaps are illiquid**

**Agreed survival probability**

Number of deaths

Initial number of lives
SIMULATED INTEREST RATE SCENARIOS

Realization of the short rate over a period of 60 years in two difference stochastic scenarios
SIMULATED MORTALITY INTENSITIES

Mortality intensities for the insurance portfolio (red lines) and the population (blue lines) in two stochastic scenarios (left plot and right plot).
Deterministic mortality intensities with a trend of decline (black and grey lines)
Deaths in the insurance portfolio (left plot) and deaths in the population (right plot) in the two scenarios
SIMULATED SURVIVOR SWAP PRICE PROCESSES

The hedging instruments
Intrinsic value processes for survivor swap on the insurance portfolio (left plot) and survivor swap on the population (right plot) in the two scenarios
The liability - to be hedged!

Intrinsic value processes for the insurance contract in the two scenarios. Example: Age 30, Life annuity starting at age 60; yearly premiums $n_1=100$, $n_2=1000$, 
SIMULATED OPTIMAL SURVIVOR SWAPS (OWN PORTFOLIO)

Number of survivor swaps on the insurance portfolio held at time $t$ in the market $(B, P, Z_1)$

Perfect hedge after retirement
SIMULATED OPTIMAL SURVIVOR SWAPS (POPULATION)

Number of survivor swaps on the population held at time $t$ in the market $(B, P, Z_2)$

Swaps on population are not useful here due to short time horizon. Main risk is unsystematic risk.
ILLIQUID SURVIVOR SWAPS: COMPARISON OF DYNAMIC AND STATIC TRADING STRATEGIES

Trading at time 0 and time 30

Swaps on insurance portfolio

Dynamic trading

Swaps on population portfolio

Trading at time 0 and time 30

Dynamic trading

Number of trading times is important
## EFFICIENCY

### Comparison of initial intrinsic risk

<table>
<thead>
<tr>
<th>n₁</th>
<th>n₂</th>
<th>No swap</th>
<th>D₁</th>
<th>S₁.1</th>
<th>S₁.2</th>
<th>S₂.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.111</td>
<td>0.048</td>
<td>0.105</td>
<td>0.073</td>
<td>0.104</td>
</tr>
<tr>
<td>100</td>
<td>10,000</td>
<td>0.111</td>
<td>0.048</td>
<td>0.105</td>
<td>0.073</td>
<td>0.100</td>
</tr>
<tr>
<td>1,000</td>
<td>10,000</td>
<td>0.062</td>
<td>0.032</td>
<td>0.045</td>
<td>0.038</td>
<td>0.039</td>
</tr>
<tr>
<td>1,000</td>
<td>100,000</td>
<td>0.062</td>
<td>0.032</td>
<td>0.045</td>
<td>0.038</td>
<td>0.037</td>
</tr>
<tr>
<td>10,000</td>
<td>100,000</td>
<td>0.055</td>
<td>0.013</td>
<td>0.022</td>
<td>0.019</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Good results even with few trading times
**CONCLUSIONS**

**Survivor swaps:**

- Match sensitivity towards mortality and longevity risk
- Optimal number is a weighted assessment of systematic and unsystematic mortality risk
- Size of hedging portfolio matters
  - Small external hedging portfolios introduce new unsystematic risk and will not be efficient as hedging instrument
- Swap on own portfolio or on (other) total population:
  - Own portfolio most efficient – may be expensive. Less liquid
  - Other large reference portfolio can work very well. More liquid

Compared dynamic to mixed dynamic and static strategies

Intrinsic risk is almost unaffected by restriction to static strategies
# RELATED RESEARCH ON MORTALITY MODELING AND MORTALITY RISK MANAGEMENT

## HEDGING WITH SURVIVOR SWAPS


## MORTALITY RISK AND MORTALITY DERIVATIVES


## DYNAMIC HEDGING WITH MORTALITY DERIVATIVES


## VALUATION AND HEDGING WITH TRADITIONAL BONDS